Statistical Interpolation Analysis Equations

$$\mathbf{w}^a = \mathbf{w}^f + \mathbf{K}\left(\mathbf{w}^o - \mathbf{H}\mathbf{w}^f\right) \tag{1}$$

$$\mathbf{K} = \mathbf{P}^f\mathbf{H}^T\left(\mathbf{H}\mathbf{P}^f\mathbf{H}^T + \mathbf{R}\right)^{-1}. \tag{2}$$

- $\mathbf{H}$ = interpolation from model grid to observation locations.
- $\mathbf{P}^f$ = first-guess error covariance matrix.
- $\mathbf{R}$ = 0bservation error covariance matrix.

Models of $\mathbf{P}^f$

Analytical models.

$\mathbf{P}^f$ comes from an analytic correlation function e.g., a Gaussian

$$\mathbf{G}^f_{ij} = \sigma_f^2 \exp\left(-\frac{(\mathbf{x}_i - \mathbf{x}_j)^2}{L_{\mathbf{x}}^2} - \frac{(\mathbf{y}_i - \mathbf{y}_j)^2}{L_{\mathbf{y}}^2}\right)$$

σ_f, $L_{\mathbf{x}}$, $L_{\mathbf{y}}$ estimated in some fashion, e.g. ML estimation from innovations.

Reduced-space models.

Reduced-space Kalman filter, ensemble Kalman filer. P^f has low-rank

$$\mathsf{P}^f = \sum_{i=1}^{m} \sum_{j=1}^{m} \mathsf{F}_{ij} \mathbf{z}_i \mathbf{z}_j^T = \mathsf{ZFZ}^T.$$

$\mathbf{z}_i$ basis of reduced space. $\mathsf{Z} = [\mathbf{z}_1, \ldots, \mathbf{z}_m]$.

F evolves dynamically or is estimated.

Simple idea: combine the two.

$$\mathbf{P}^f = \mathbf{G}^f_\perp + \mathbf{ZFZ}^T = (\mathbf{I} - \mathbf{ZZ}^T)\mathbf{G}^f(\mathbf{I} - \mathbf{ZZ}^T) + \mathbf{ZFZ}^T.$$

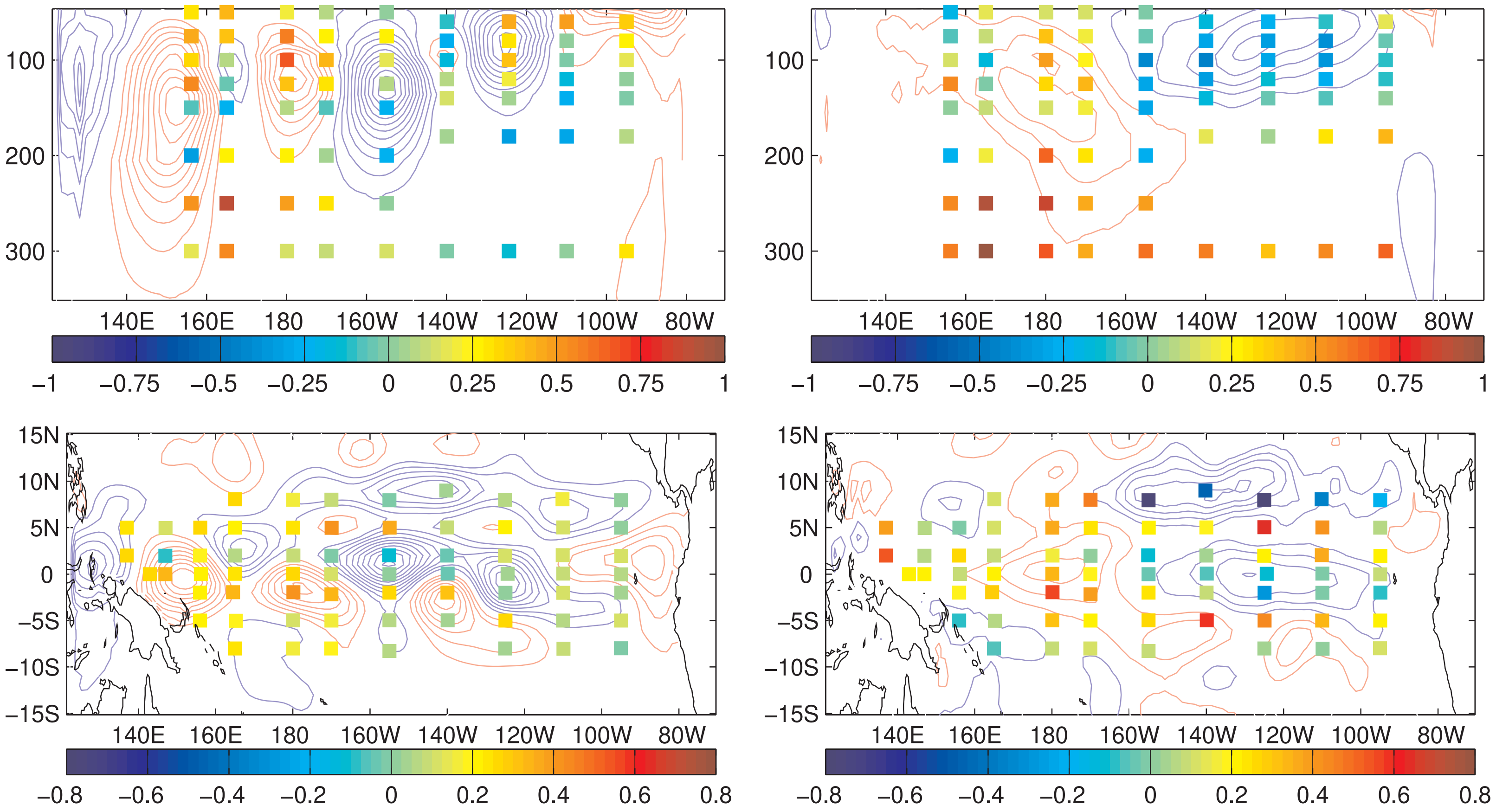
Implementation

- NCEP Pacific basin version of MOM-1.
- EOFs come from a long forced run.
- $\mathbf{F} = \alpha \mathbf{I}$.
- Jan 1993 - Feb 1997.

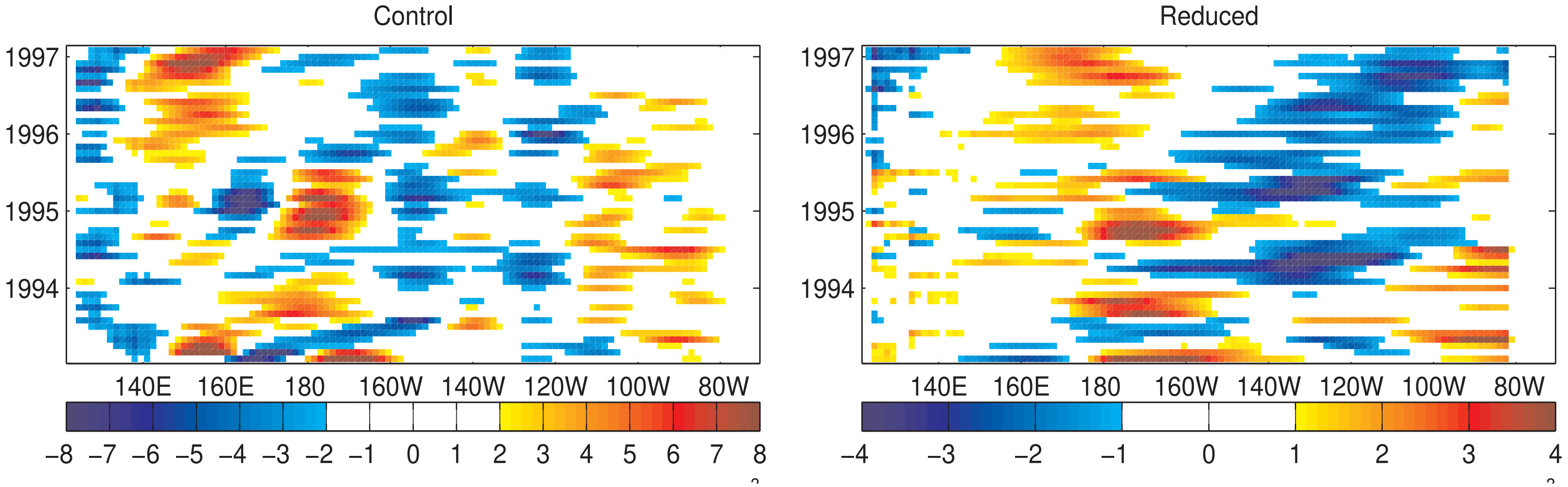
Impacts.

- Temperature correction.
- Mean temperature. y
- Zonal current.

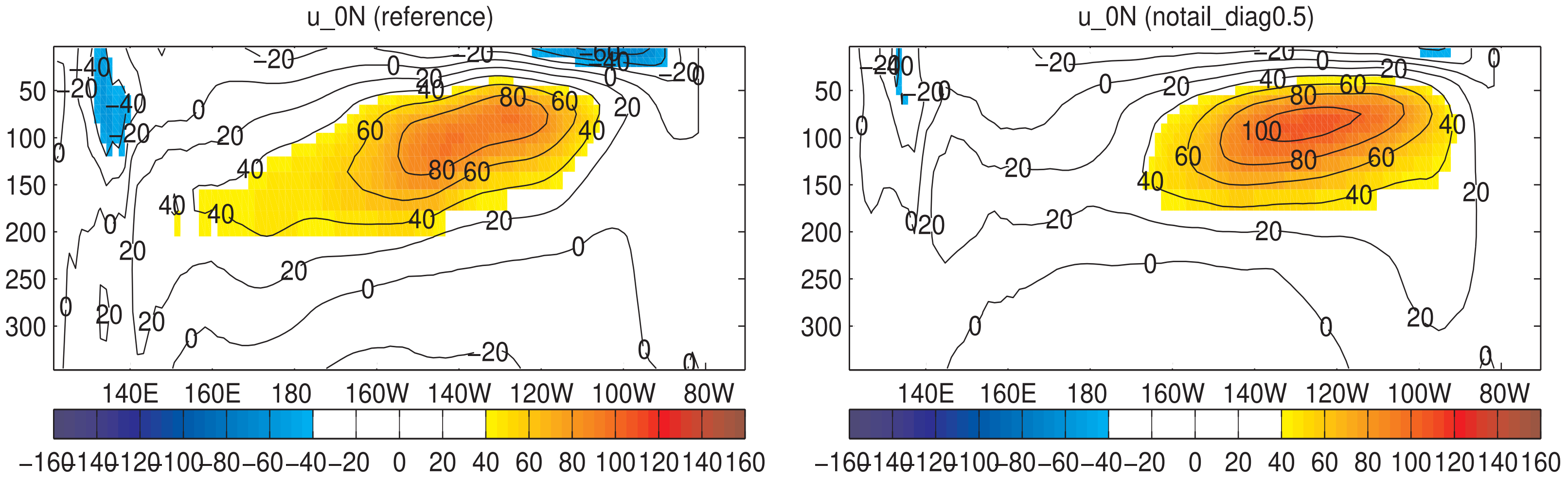
Temperature corrections



Vertically averaged equatorial temperature corrections.



Zonal currents



Mean equatorial temperature.