Filtering of GCM simulated Sahel precipitation

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[1] Atmospheric general circulation models (GCMs) forced with observed sea surface temperature (SST) reproduce some aspects of observed Sahel rainfall variability, particularly decadal variability. Here a filter based on signal-to-noise (S/N) EOFs is applied to seven GCM simulations of Sahel precipitation to extract SST-forced variability. Using filter coefficients based on GCM estimates of internal variability has limited, though positive, impact on simulation skill. Additional removal of empirically identified model error improves the representation of both decadal and interannual variability. The model error shows some coherence across the seven GCMs and correlates with local Atlantic SST. We hypothesize that the model error is related to the representation of ocean-atmosphere interactions in the SST-forced GCM simulations. Citation: Tippett, M. K. (2006), Filtering of GCM simulated Sahel precipitation, Geophys. Res. Lett., 33, L01804, doi:10.1029/2005GL024923.

1. Introduction

[2] Sahelian rainfall shows considerable variability on both interannual and decadal time-scales, including several decades of dry conditions beginning in the late 1960s and continuing into the 1990s. Sea surface temperature (SST) is one of the modulators of Sahelian rainfall, and statistical methods have identified SST patterns related to Sahelian rainfall variability [Folland et al., 1986]. General circulation models (GCMs) provide a dynamical estimate of the atmospheric response to SST forcing [Rowell et al., 1995]. Many GCMs forced by observed SST reproduce aspects of the decadal variability of Sahelian rainfall, but most have difficulty reproducing interannual variability [Giannini et al., 2003; Moron et al., 2003].

[3] The deficiencies of GCM simulations could be due to an inherent lack of predictability, in the sense that SST information alone may be insufficient to constrain Sahelian rainfall interannual variability. However, this explanation seems questionable since at least one GCM forced with observed SST reproduces Sahelian rainfall interannual variability [Giannini et al., 2003, 2005]. Poor representation of physical processes, particularly those related to convection, may be a problem in the GCM simulations. Statistical methods can be used to compensate for such model errors when GCMs successfully simulate part of the climate signal [Feddersen et al., 1999; Tippett et al., 2005], and GCM simulated winds are a useful surrogate for observed Sahelian rainfall [Moron et al., 2004]. Another potential source of error in GCM simulations forced by observed SST is the neglected feedback of the atmosphere to the ocean [Peña et al., 2003] which has been shown to be important in the simulation of the Asian monsoon [Fu et al., 2002; Wang et al., 2004].

[4] Here a filtering approach is used to reduce the effects of atmospheric internal variability and model error in seven GCM simulations of Sahelian precipitation. Using ensemble averages and projecting the GCM output onto signal-to-noise (S/N) EOFs reduces the impact of internal variability [Hasselmann, 1979, 1997; Venkatesh et al., 1999; Barreiro et al., 2002]. The S/N EOFs maximize the ensemble mean variance relative to the inter-ensemble variance and are the most reproducible modes of the GCM given the SST forcing; S/N analysis of one GCM identified modes whose time-series correlated well with Sahelian rainfall variability on decadal and interannual time-scales [Tippett and Giannini, 2006]. A least-squares estimate of the SST-forced signal is constructed by applying a damping factor in conjunction with the projection of the ensemble mean onto S/N EOFs. The damping factor depends on the ensemble size and S/N ratio of each mode; larger S/N ratio and ensemble size leads to less damping. The filter mimics the reduction of internal variability obtained by ensemble averaging. However, as shown in Section 3, direct use of this filtering method does not lead to substantial improvement of the interannual skill of the GCM simulations of Sahelian rainfall. Modifying the filter to remove model error improves the representation of Sahelian rainfall variability on both decadal and interannual time-scales. Model error is classified as those simulation components that, despite being identified by the S/N analysis as robust GCM responses to SST forcing, do not contribute to simulation skill. Some commonality is found in the model error across GCMs.

2. Data

[5] All analyses use July–September (JAS) seasonal averages. The seven GCMs analyzed are: ECHAM 4.5, GFDL AM2p12b, NSIPP-1, ECPC, COLA, CCM 3.6, and NCEP/MRF9 [Roeckner et al., 1996; GFDL Global Atmospheric Model Development Team, 2004; Bacmeister et al., 2000; Kanamitsu et al., 2002; Kinter et al., 1997; Hack et al., 1998; Livezey et al., 1996, respectively]. The GCMs are forced with observed SST; different SST analyses were used. The simulation period, number of ensemble members and horizontal resolution for each GCM are shown in Table 1.

[6] The S/N EOFs for each GCM are computed using simulated precipitation over the West Africa region 2°N to 20°N and 20°W to 35°E containing 176 GCM grid points for the T42 resolution. The primary rainfall observations used in this analysis come from the Hulme precipitation data set, based on gauge data gridded at 2.5° latitude by

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Table 1. Simulation Period, Ensemble Size and Horizontal Resolution of the GCMs Used in This Study

<table>
<thead>
<tr>
<th>Model</th>
<th>Period</th>
<th>Ensemble Size</th>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECHAM 4.5</td>
<td>1950–2004</td>
<td>24</td>
<td>T42</td>
</tr>
<tr>
<td>GFDL AM2p12b</td>
<td>1950–2000</td>
<td>10</td>
<td>2.5° × 2.5°</td>
</tr>
<tr>
<td>NSIPP-1</td>
<td>1950–2000</td>
<td>9</td>
<td>2.5° × 2.5°</td>
</tr>
<tr>
<td>ECPL</td>
<td>1950–2001</td>
<td>10</td>
<td>T63</td>
</tr>
<tr>
<td>COLA</td>
<td>1950–2003</td>
<td>10</td>
<td>T63</td>
</tr>
<tr>
<td>CCM 3.6</td>
<td>1950–2004</td>
<td>24</td>
<td>T42</td>
</tr>
<tr>
<td>NCEP MRF9</td>
<td>1950–1999</td>
<td>10</td>
<td>T42</td>
</tr>
</tbody>
</table>

3.75° longitude resolution for the 49-year period 1950–1998 [Hulme, 1992]; additional analysis uses the CPC Merged Analysis of Precipitation (CMAP) [Xie and Arkin, 1996]. SST data are taken from the ERSST data set version 2 [Smith and Reynolds, 2004].

3. Methods

A least-squared error estimate of the SST-forced signal is the starting point for the derivation of the filter. Suppose the GCM precipitation anomaly field x (an n-dimensional column vector) of a particular ensemble member is decomposed

\[ x = x_S + x_N, \]  

where \( x_S \) is the SST-forced signal and \( x_N \) represents random internal variability (“noise”) unrelated to the SST forcing. The ensemble mean \( M \) of an \( m \)-member ensemble has a smaller contribution from internal variability and can be written

\[ x_M = x_S + \frac{x_N}{\sqrt{m}}. \]  

The least-squares estimate \( \hat{x}_S \) of the signal from the ensemble mean \( x_M \) is given by linear regression as

\[ \hat{x}_S = \langle x_S x_M^T \rangle_M (x_M x_M^T)^{-1} x_M, \]  

where the notation \( (\cdot)^T \) and \( \langle \cdot \rangle_M \) denotes transpose and expectation, respectively. Defining the covariances of the ensemble mean and internal variability \( C_M \equiv \langle x_M x_M^T \rangle_M \) and \( C_N \equiv \langle x_N x_N^T \rangle_M \), respectively, it follows from (3) that \( \langle x_S x_M^T \rangle_M = C_M - C_N/m \), and the estimate \( \hat{x}_S \) of the signal is

\[ \hat{x}_S = \left( I - \frac{1}{m} C_N C_M^{-1} \right) x_M, \]  

where \( I \) is the identity matrix.

S/N analysis can be used to interpret the regression in (4) as a filter, and to show that the regression matrix \( I - C_N C_M^{-1}/m \) is diagonal in the basis of S/N EOFs; this interpretation is new to the author’s knowledge. Each S/N EOF is associated with a vector of coefficients \( f \) (an optimal spatial filter in the terminology of Venzke et al. [1999]), a physical pattern \( p \) and a S/N ratio \( \lambda \). The leading S/N EOF maximizes the S/N ratio

\[ \lambda = \frac{f^T C_M f}{f^T C_N f}. \]  

of the linear combination \( f^T x_M \); regressing the time series of the linear combination onto the ensemble mean gives the physical pattern \( p \). Subsequent (in order of decreasing S/N ratio) S/N EOFs maximize the S/N ratio under the constraint that the time-series be uncorrelated; unlike usual EOFs, S/N EOFs are not orthogonal in space. The coefficient vectors are eigenvectors of the matrix \( C_N^{-1} C_M \), whose eigenvalue decomposition is \( C_N^{-1} C_M = F \Lambda F^{-1} \) where the columns of \( F \) are coefficient vectors and the entries of the diagonal matrix \( \Lambda \) are the associated S/N ratios. Associated with the matrix \( F \) of coefficients is the matrix \( P \) of patterns defined by \( P = F^{-T} \); the pseudo-inverse is used for the rank deficient problem. Since \( C_N^{-1} C_M = P \Lambda^{-1} P^T \), the optimal estimate in (4) of the SST-forced signal can be expressed using S/N EOFs as

\[ \hat{x}_S = P (I - (m \Lambda)^{-1}) P^T x_M, \]

\[ = \sum_{i=1}^{n} p_i \left( 1 - (m \Lambda_i)^{-1} \right) f_i^T x_M, \]

where \( f_i \) and \( p_i \) are the \( i \)-th columns of the matrices \( F \) and \( P \) respectively, and \( \Lambda_i \) the diagonal elements of the matrix \( \Lambda \); the regression matrix in (4) is diagonal in the basis of the columns of \( P \). Equation (6) shows the signal estimate is obtained by (i) forming the linear combination \( f_i x_M \) of the ensemble mean, (ii) multiplying by a damping factor that depends on the S/N ratio and ensemble size, and (iii) reconstructing the field using the patterns. Small S/N ratio and small ensemble size lead to larger damping. No damping is necessary in the limits of large S/N ratio or large ensemble size, in which case \( PF^T = I \) implies \( \hat{x}_S = x_M \). Terms can be omitted from the sum or the damping coefficient set to zero when the S/N EOF represents an unphysical or erroneous response of the GCM to SST forcing.

Estimation of the S/N EOFs requires inverting the noise covariance \( C_N \) which is invertible for the domain size, ensemble size and number of years used; the noise is approximated by the deviations between the ensemble members and the ensemble mean. The inversion of the noise matrix \( C_N \) is computed from its eigenvalue decomposition. Since the smallest eigenvalues of the noise covariance are likely underestimated, an approach from ridge regression is used and the leading (in decreasing order) 80% of the eigenvalues are kept at their estimated value and the remaining eigenvalues set to the value of the eigenvalue at the 80% limit. This procedure has the effect of inflating the intra-ensemble spread associated with the smallest eigenvalues of the noise covariance matrix \( C_N \).

4. Results

The results are summarized using a Sahel rainfall index defined as the average of the precipitation anomalies in the box 16°W to 30°E and 10°N to 20°N. This index is computed for the observations and for three sets of GCM data: the raw ensemble mean (denoted raw) and two filter experiments. The F1 filter experiment retains modes in (6) with a S/N ratio greater than 1.134; this value corresponds to a perfect model correlation of 0.75. The F2 filter experiment keeps modes that have a S/N ratio greater than
Correlations of the Simulation (Unfiltered and Two Types of Filters) of Each of the Seven GCMs and the Multi-Model Sum
With Observed Precipitation and the ENSO State*

<table>
<thead>
<tr>
<th>Model</th>
<th>Modes (F2)</th>
<th>% Variance (F2)</th>
<th>(r_{\text{total}}) Raw</th>
<th>(r_{\text{total}}) F1</th>
<th>(r_{\text{total}}) F2</th>
<th>(r_{\text{hf}}) Raw</th>
<th>(r_{\text{hf}}) F1</th>
<th>(r_{\text{hf}}) F2</th>
<th>(r_{\text{hf}}) Raw</th>
<th>(r_{\text{hf}}) F1</th>
<th>(r_{\text{hf}}) F2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECHAM 4.5</td>
<td>1,2,3,4</td>
<td>70</td>
<td>0.53</td>
<td>0.73</td>
<td>0.83</td>
<td>-0.076</td>
<td>0.31</td>
<td>0.7</td>
<td>0.031</td>
<td>-0.26</td>
<td>-0.5</td>
</tr>
<tr>
<td>GFDL AM2p12b</td>
<td>1,2,3,4,5,7,8</td>
<td>64</td>
<td>0.6</td>
<td>0.6</td>
<td>0.78</td>
<td>0.14</td>
<td>0.18</td>
<td>0.6</td>
<td>-0.13</td>
<td>-0.14</td>
<td>-0.35</td>
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<tr>
<td>NSIPP-1</td>
<td>1,2,3,4,9</td>
<td>65</td>
<td>0.79</td>
<td>0.81</td>
<td>0.85</td>
<td>0.49</td>
<td>0.51</td>
<td>0.6</td>
<td>-0.51</td>
<td>-0.52</td>
<td>-0.58</td>
</tr>
<tr>
<td>ECPC</td>
<td>1,2,4,5,8,9,10</td>
<td>55</td>
<td>0.77</td>
<td>0.77</td>
<td>0.79</td>
<td>0.4</td>
<td>0.45</td>
<td>0.65</td>
<td>-0.63</td>
<td>-0.65</td>
<td>-0.64</td>
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<tr>
<td>COLA</td>
<td>1.2</td>
<td>40</td>
<td>0.58</td>
<td>0.57</td>
<td>0.85</td>
<td>0.027</td>
<td>-0.0031</td>
<td>0.68</td>
<td>-0.032</td>
<td>-0.017</td>
<td>-0.3</td>
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<td>CCM 3.6</td>
<td>4</td>
<td>7.5</td>
<td>0.35</td>
<td>0.33</td>
<td>0.61</td>
<td>0.048</td>
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<td>-0.23</td>
<td>-0.16</td>
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<tr>
<td>NCEP MRF9</td>
<td>1,4,7,8</td>
<td>55</td>
<td>0.65</td>
<td>0.68</td>
<td>0.74</td>
<td>0.14</td>
<td>0.2</td>
<td>0.37</td>
<td>-0.39</td>
<td>-0.4</td>
<td>-0.38</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum-50-98</td>
<td>0.72</td>
<td>0.76</td>
<td>0.23</td>
<td>0.3</td>
<td>0.75</td>
<td>-0.31</td>
<td>0.69</td>
<td></td>
<td>-0.35</td>
<td>-0.41</td>
<td>-0.59</td>
</tr>
<tr>
<td>Sum-79-98</td>
<td>0.72</td>
<td>0.76</td>
<td>0.23</td>
<td>0.3</td>
<td>0.75</td>
<td>-0.31</td>
<td>0.69</td>
<td></td>
<td>-0.35</td>
<td>-0.41</td>
<td>-0.59</td>
</tr>
</tbody>
</table>

*aSee text for details.

1.134 and that improve the correlation (high frequency) with the observed Sahel index.

[11] Correlations between the observations and the GCM indices are shown in Table 2; also included are correlation with the standardized multi-model sum of the GCM timeseries. The notation \(r_{\text{total}}\) denotes the correlation with the observed index, and \(r_{\text{hf}}\) denotes the correlation of the high-frequency components, defined as the deviation from the 11-year running-average. The correlation \(r_{\text{total}}\) of the raw GCM output of most of the models is high; \(r_{\text{total}} = 0.72\) for the multi-model sum. However, much of that correlation is related to decadal variability, and only two GCMs have a significant high-frequency correlation with the observations; \(r_{\text{hf}} = 0.31\) for the multi-model sum. Figure 1a shows that the GCM high-frequency indices are more similar to each other than to the observations; the correlation of the multi-model sum with the individual models, a perfect model skill measure, is 0.69. The raw indices of the GCMs with high good-frequency simulation skill correlate appropriately with NINO 3.4 (see Table 2); the observed correlation between the Sahel index and NINO 3.4 is \(-0.59\).

[12] The F1 filter results show a generally positive impact on the correlation \(r_{\text{total}}\) and a modest positive impact on \(r_{\text{hf}}\). In the F2 filter, modes that improve the correlation \(r_{\text{hf}}\) are retained, explaining from 7.5 to 70 percent of the GCM precipitation variance in the entire West Africa domain (see Table 2). Filter F2 shows increased values of the total correlation \((r_{\text{total}} = 0.88\) for the multi-model sum) as well as increased values of the high-frequency correlation \((r_{\text{hf}} = 0.75\) for the multi-model sum; see Figure 1b). The high-frequency correlation of the multi-model sum of the F2 filter indices with NINO 3.4 is \(-0.59\), very similar to the observed value. To measure the robustness of the F2 filter, we classified model error modes based on 1950–1978 performance, and calculated the F2 filter performance for periods 1950–1998 and 1979–1998 (multi-model sums labeled Sum-50-98 and Sum-79-98, respectively, in Table 2). This procedure gives a sense of the model error estimate sensitivity as well as stationarity of the GCM skill. Overall, the F2 filter still shows improvement though the skill of some GCM simulations is degraded; GCMs where F2 explained little variance or had modest skill were most strongly affected.

[13] Model error is defined for each GCM as the difference between the F1 and F2 filtered simulations and summarized by the associated standardized high-frequency Sahel index time-series (see Figure 2a). Although there is considerable variability across models, the correlation of the multi-model sum of the model error indices with that of the individual models is 0.6, and there is consensus in the sign of the model error in some years. Correlation of SST with the multi-model sum of the model error indices (Figure 2b) shows small positive (maximum value \(-0.4\) correlations off the coast of West Africa. The sign of the correlation means that warm SST near the West African coast is associated with a positive error in the GCM simulated Sahel index. A large positive value of the mean model error index occurs in JAS 1984, one of the driest of the drought years, when several of the GCMs overestimated Sahel precipitation (Figure 2a); the error of 1984 was also observed in the French model ARPEGE and discussed by Garric and Déqué [2002]. The 1984 SST was anomalously warm in the Gulf of Guinea and off the coast of Senegal, exceeding 27°C (see Figures 3a and 3b). CMAP estimates show positive rainfall anomalies in the Guinea coast region (consistent with the warm SST anomalies in the Gulf of Guinea) and negative rainfall anomalies in the Sahel region and off the coast of Senegal (see Figure 3c). However, the
GCMs, while simulating dry conditions in part of the Sahel region, simulated wetter conditions on the coast near the warm SST anomalies; the precipitation anomaly of the ECPC GCM is shown in Figure 3d. One possible explanation for the error in the GCM Sahel simulation is that the response to SST forcing in the Guinea Gulf is too strong and affects the Sahel region. Also, the GCMs may be responding to warm SST near the coast of Senegal. The negative precipitation anomalies over ocean in the CMAP estimate are based on satellite data and may reflect decreased cloudiness that may be related to the coincident positive SST anomalies. Also reduced Sahel rainfall is associated with weak monsoon flow, driving less oceanic upwelling in the Guinea Dome region and resulting in warmer local SST [Fontaine et al., 1995; Signorini et al., 1999]. Either mechanism gives SST that is negatively correlated with local precipitation which is difficult to represent in the SST-forced GCM simulations.

5. Summary

[14] Many GCM simulations of Sahelian rainfall reproduce aspects of decadal variability observed in the 20th century. Fewer models, however, reproduce the observed interannual variability. Here, a signal-to-noise (S/N) filtering technique has been applied to the precipitation simulations of seven GCMs forced by observed SST. There is little improvement when the S/N ratio is used to determine filter coefficients. Significant improvement is seen when modes classified as model error are removed.

[15] Model error, defined as comprising those components that degrade simulation skill in spite of significant S/N ratio, shows some coherence across the seven GCMs. The multi-model model error time-series shows correlation with regional Atlantic SST, suggesting that the efficacy of using the S/N EOFs as a filter basis due to signal and error being related to SST. In 1984, when several GCMs overestimated Sahel rainfall, warm SST conditions were observed near the coast of West Africa. The enhanced SST near the coast of Senegal, possibly due to reduced cloudiness or oceanic upwelling, appears to be associated with spurious enhanced precipitation in the GCMs. We hypothesize that part of the systematic model error observed in GCM simulations of Sahel rainfall is related to error in the representation of ocean-atmosphere interaction in SST-forced simulations.

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