Environmental Quality, Environmental Protection and Technology Adoption

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Abstract

Several empirical studies support the environmental Kuznets curve hypothesis (EKC) for some pollutants – as income increases pollution increases, reaches a peak and eventually starts to decrease. Despite relying mostly on cross-sectional data for different countries, these studies tend to interpret the EKC from a time-series perspective: the EKC is a by-product of economic growth. This paper formally puts forth a qualification to the common interpretation that the EKC is a by-product of economic growth by investigating how barriers to technology adoption affect environmental quality in different countries. Barriers to technology adoption account for much of the variation in total factor productivity (TFP) and income across countries, and this paper investigates the effect of these barriers on the environment and the relationship between development and environmental quality. It does so by performing comparative statics on the steady state of a dynamic economy, thus analyzing the effect of changes of the economy’s TFP on environmental quality. The model also enables investigation of the effect of barriers to technology adoption on pollution rates and environmental protection expenditures. (JEL O13, Q20)

Keywords: Environmental Quality, Barriers to Technology Adoption, TFP.
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1 INTRODUCTION

This paper investigates how barriers to technology adoption affect environmental quality in different countries. More specifically, this paper shows how different technologies contribute to a U-shaped relationship between environmental quality and income in cross-sections of countries. This relationship is implied by the environmental Kuznets curve (EKC), an inverted U-shaped relationship between pollution and income. We conduct the analysis by focusing on Pareto optimality in the steady state of a dynamic economy where different technologies are considered.

A number of studies investigate empirical patterns of pollution (and implied environmental quality) at different levels of income. In cases where pollution control is technically and institutionally feasible, the EKC indicates that emissions tend to rise with income up to a point where they start to decline (see for example, Shafik and Bandyopadhyay, 1992, Grossman and Krueger, 1995, and Selden and Song, 1994). The EKC is often interpreted as a by-product of economic growth, implying that the decline and subsequent recovery of environmental quality is a matter of time reflecting the natural path of economic development. For example, Grossman and Krueger (1995, p. 372) state that “(...) air and water quality appear to benefit from economic growth once some critical level of income has been reached.” In a similar empirical paper, Selden and Song (1994, p. 147) write that “(...) it is reasonable to expect that economies would pass through ‘stages of development’, in which at least some aspects of environmental quality first deteriorate and then improve.” However, since time
series on pollution and environmental quality are generally short and variable in quality, evi-
dence on the relationship between environmental quality and economic development heavily relies on cross-sectional data for different countries. The use of cross-sectional data raises the question of whether country-specific characteristics matter when explaining the EKC. If this is the case, as this paper suggests, the time-series interpretation bears an extra burden of proof, since it assumes that countries are identical and follow a predetermined path for environmental quality.

Most empirical studies on development and the environment use panel data analysis. For example, Grossman and Krueger (1995) and Selden and Song (1994) use panel data to investigate how pollution responds to income, and they find an inverted-U relationship between these variables. With panel data analysis, the effect of country specific characteristics can be explored by estimating within or random effects models. However, in the case where the country specific characteristics are correlated with income (the usual measure for economic development), the estimates of the relationship between environmental quality and economic development are subject to bias and inconsistency (Hausman and Taylor, 1981). This paper provides theoretical support to these qualifications by considering barriers to technology adoption in different countries.

Barriers to technology adoption constitute an important factor that is believed to account for much of the variation in income across countries (Parente and Prescott, 1994 and 2000). This paper investigates the effect of these barriers on the environment and their contribution to the empirical relationship between development and environmental quality. It does so by performing comparative statics on the steady state of a dynamic economy where the society’s total factor productivity (TFP) is allowed to change. Heterogeneous TFPs across countries correspond to different multiplicative technological parameters of their aggregate production
functions (Parente and Prescott, 2000). Furthermore, the model presented here enables investigation of the effect of barriers to technology adoption in two other aggregate technical relationships: (i) the aggregate environmental protection function interpreted as end-of-the-pipe environmental clean-up; and (ii) the aggregate pollution function, assumed to depend on the stock of capital of the economy.

2 RELATED LITERATURE

After the initial studies highlighting the EKC in the early 1990s, several scholars have tried to provide a theoretical explanation for the phenomenon, mostly considering the dynamic nature of pollution (and environmental quality). In a simple dynamic model, Selden and Song (1995) derived conditions for the EKC that are sufficient but not necessary. John and Pecchenino (1994) and Jones and Manuelli (1994) used overlapping generations models where the young choose a tax scheme that accounts for environmental quality when they are old. In a more recent paper, Stokey (1998) developed a dynamic model where the EKC is consistent with Pareto optimality, although the result depends on the rather restrictive assumption that more efficient technologies are necessarily more pollution intensive. In a different context, Reppelin-Hill (1999) analyzed the case where a more efficient technology is less pollution intensive. Two static models consistent with the EKC appear in Andreoni and Levinson (1998) and Stokey (1998).

These studies fail to recognize that an important cause of the great differences in national incomes is heterogeneity in their total factors of productivity (TFPs) as pointed out by Parente and Prescott (1994). The difference in TFPs originates from a nation's institutional environment, such as regulatory and legal constraints, cultural values, corruption, violence, sabotage,
and worker strikes (Parente and Prescott, 1994). These institutional factors help determine the optimal stock of capital, environmental quality and aggregate output in each country and are essential in the analysis of the relationship between environmental quality and income, especially when most empirical evidence on this relationship relies on cross-country observations. For example, large barriers to technology adoption reduce capital productivity, thus leading to a smaller capital stock, aggregate output and pollution. In this scenario, the shadow value of capital is likely to exceed that of environmental quality. A reduction in barriers to technology adoption permits more capital utilization and pollution. As barriers to technology adoption are further reduced, increased capital accumulation and wealth cause the shadow values to equalize. Further growth will enable both further capital accumulation and increasing environmental protection.

In addition to explaining the effect of differences in TFPs, this paper analyzes the effect of heterogeneity in technical parameters of the aggregate environmental protection and pollution functions of different nations. The differences in these technical parameters also originate from the institutional environment and have an impact on environmental quality, environmental expenditures, consumption and capital accumulation.

3 MODEL

Consider a dynamic model with environmental quality treated as a stock variable. That is, environmental quality in any time period depends on cumulative pollution and environmental protection. Leading examples of environmental phenomena best characterized as stocks include depletion of the ozone layer, the greenhouse effect, and deforestation and biodiversity loss.
In the dynamic model presented here, the social planner maximizes the stream of social welfare over an infinite time horizon. Each individual in society values consumption per capita \( c \) and the stock of environmental quality \( E \) at each time \( t \). Welfare is defined as the summation of the utility functions \( u(c, E) \) of \( N \) representative individuals and its maximization is constrained by the laws of motion for the stock of capital \( (K) \) and environmental quality \( (E) \). Capital accumulation results from the difference between total production \( (F(K)) \), a concave function of capital) and aggregate expenditure on consumption \( (Nc) \) and environmental preservation effort \( (\pi) \), both measured in units of output. For simplicity, assume zero capital depreciation. Environmental quality on the other hand decreases with the stock of capital (pollution function, \( P(K) \)) and increases with effort on environmental protection (environmental improvements function, \( \Pi(\pi) \)). The problem is formally described as follows:

\[
\max_{c, \pi} \int_0^\infty e^{-\rho t} Nu(c, E)dt
\]

subject to:

\[
\dot{E} = -P(K) + \Pi(\pi), \quad \dot{K} = F(K) - Nc - \pi, \quad E \geq 0, \quad K \geq 0,
\]

\[
c \geq 0, \quad \pi \geq 0, \quad E(0) = E_0, \quad K(0) = K_0,
\]

where \( \rho \) is the real discount rate \((\rho > 0) \). Also, assume that:

\[
u_c > 0; \ u_{cc} < 0; \ u_E > 0; \ u_{EE} < 0; \ u_{cE} \geq 0; \ \lim_{c \to 0} u_c = \infty; \ \lim_{E \to 0} u_E = \infty,
\]

\[
P_K > 0; \ P_{KK} > 0; \ \lim_{K \to 0} P_K = 0; \ \lim_{K \to \infty} P_K = \infty,
\]

\[
F_K > 0; \ F_{KK} < 0; \ \lim_{K \to 0} F_K = \infty; \ \lim_{K \to \infty} F_K = 0,
\]

\[
\Pi_{\pi} > 0; \ \Pi_{\pi\pi} \leq 0; \ \lim_{\pi \to 0} \Pi_{\pi} = \infty; \ \lim_{\pi \to \infty} \Pi_{\pi} = 0.
\]
3.1 Optimality

The current value Hamiltonian for an interior solution is given by:

\[
H = Nu(c, E) + \lambda[-P(K) + \Pi(\pi)] + \mu[F(K) - Nc - \pi].
\]

The necessary conditions for the above problem are:

\[
\frac{\partial H}{\partial c} = Nu_c - N\mu = 0 \implies \mu = u_c,
\]

\[
\frac{\partial H}{\partial \pi} = \lambda\Pi_\pi - \mu = 0 \implies \lambda = \frac{\mu}{\Pi_\pi},
\]

\[
\dot{\lambda} = \rho\lambda - Nu_E u_c,
\]

\[
\dot{\mu} = \rho\mu - [-\lambda P_K + \mu F_K].
\]

Manipulation of the necessary conditions yields:

\[
\dot{c} = \frac{u_c}{u_{cc}} \left( \frac{P_K}{\Pi_\pi} - F_K + \rho - \frac{u_c E}{u_c} \dot{E} \right), \quad (1)
\]

\[
\dot{\pi} = \frac{\Pi_\pi}{\Pi_{\pi\pi}} \left( \frac{u_c c + u_c E \dot{E}}{u_c} - \rho + N \frac{u_E}{u_c} \Pi_\pi \right). \quad (2)
\]

Along the optimal consumption path given by equation (1), a larger marginal contribution of capital to pollution or smaller productivity of capital reduces the rate of increase in consumption. Also, increasing environmental quality over time ($\dot{E} > 0$) accelerates consumption growth. Similarly, barriers to technology adoption that make preservation effort less productive (decrease $\Pi_\pi$) contribute to slower consumption growth.
Suppose the economy is growing. Equation (2) indicates that, ceteris paribus, if consumption per capita is increasing, so is preservation effort in order to compensate for a more degraded environment due to increasing use of capital. Similarly, if environmental quality is decreasing over time, preservation effort will grow faster to keep the discounted stream of utility at a maximum. Also, growth of preservation effort over time is decreasing in the discount rate and increasing in the marginal rate of substitution of environmental quality for consumption and marginal environmental improvement from environmental expenditures $\pi$.

### 3.2 Steady State

Important insight can be obtained by studying the steady state of this dynamic economy. The motivation for focusing on the steady state is twofold: It simplifies the analysis and, most importantly, it allows us to focus on the underlying economic forces of interest. For simplicity, we assume a constant elasticity utility function. For $\sigma, \beta, \varphi, \psi > 0$, and $0 < \delta \leq 1$, define the utility and environmental protection functions as follows:

\[
\begin{align*}
  u(c, E) &= \varphi \frac{c^{1-\sigma} - 1 + \psi E^{1-\beta} - 1}{1-\sigma}, \\
  \Pi(\pi) &= \Pi \pi^\delta.
\end{align*}
\]

Then, rewrite equations (1) and (2) in terms of the rates of growth of consumption ($\gamma_c$) and environmental protection effort ($\gamma_\pi$). Economic growth in this economy is described by these two rates of growth plus rates of change in capital stock ($\gamma_K$) and environmental quality ($\gamma_E$):

\[
\gamma_c \equiv \frac{\dot{c}}{c} = -\frac{1}{\sigma} \left( \frac{P_K}{\Pi \pi} - F_K + \rho \right),
\]
\[ \gamma_n \equiv \frac{\dot{\pi}}{\pi} = \frac{1}{(\delta - 1)} \left( \gamma_c - \rho + N \frac{u_E}{u_c} \Pi_\pi \right), \]

\[ \gamma_E \equiv \frac{\dot{E}}{E} = -\frac{P(K) + \Pi(\pi)}{E}, \]

\[ \gamma_K \equiv \frac{\dot{K}}{K} = \frac{F(K) - Nc - \pi}{K}. \]

The steady state is defined as the state where all variables grow at a constant rate. This implies that the growth rates for the variables of the model are equal to zero in the steady state:

**Proposition 1** In the steady state, the rates of growth of consumption (\(\gamma_c\)), environmental expenditures (\(\gamma_\pi\)), the stock of capital (\(\gamma_K\)) and environmental quality (\(\gamma_E\)) are equal to zero.

**Proof:** See appendix A.

Hence, optimality in the steady state reduces to:

\[ \frac{P_K}{\Pi_\pi} - F_K + \rho = 0, \]  

(3)

\[ Nu_E \Pi_\pi - u_c \rho = 0, \]  

(4)

\[ -P(K) + \Pi(\pi) = 0, \]  

(5)

\[ F(K) - Nc - \pi = 0. \]  

(6)

Rearranging equation (4) yields the dynamic Samuelson condition for the provision of environmental quality:

\[ N \frac{u_E}{u_c} \frac{1}{\rho} = \frac{1}{\Pi_\pi}. \]

That is, the discounted sum of the marginal rates of substitution of environmental quality for
consumption across all individuals must equal the marginal cost of provision of environmental quality (units of output spent per unit of additional environmental quality).

Similarly, equation (3) indicates optimality in the production sector of the economy, i.e., the optimal trade-off between the marginal social benefit of capital and its marginal social cost. The marginal product of capital must equal the discount rate plus the cost of foregone environmental quality due to additional capital use. Clearly, given the concavity assumption on the production function $F(K)$, optimality with polluting capital implies a smaller steady state capital stock than otherwise:

$$F_K = \rho + \frac{P_K}{\Pi_\pi}.$$

**3.3 Effect of Technology Adoption on Economic Variables**

This section analyzes the effect of technological differences on the steady state of the model. We assume that this heterogeneity is due to barriers to technology adoption. Barriers to technology adoption are assumed to translate into higher costs for firms to adopt a new and higher quality technology. Parente and Prescott (2000) show how these costs affect the TFP and consequently not only the market equilibrium, but also the Pareto optimal allocations. This paper focuses on Pareto optimality and extends the concept of total factor productivity from the aggregate production function to the aggregate environmental protection and the pollution functions. For example, stringency and enforcement of national environmental regulation will provide incentives for the adoption of more efficient environmental protection technologies such as abatement technologies, thus increasing marginal efficiency of environmental protection effort ($\Pi_\pi$). At the same time it will provide incentives for the adoption of less pollution intensive technologies, thus decreasing marginal pollution of capital ($P_K$). For simplicity, the analysis presented here abstracts from interactions between these barriers to
technology adoption and treats the final effect on the parameters of the aggregate production, pollution and environmental protection functions as independent.

Without loss of generality, normalize population so that $N = 1$. Next, assume that the production function ($F(K)$), the pollution function ($P(K)$) and the environmental protection function ($\Pi(\pi)$) are as follows:

\[ F(K) = A \cdot F^\circ(K) ; \quad P(K) = B \cdot P^\circ(K) ; \quad \Pi(\pi) = D \cdot \Pi^\circ(\pi), \]

where $A$, $B$ and $D$ are the aggregate technological parameters that vary across countries and, according to our assumptions in section 3, $F^\circ_K > 0$, $F^\circ_{K,K} < 0$, $P^\circ_K > 0$, $P^\circ_{K,K} > 0$, $\Pi^\circ_\pi > 0$ and $\Pi^\circ_{\pi,\pi} \leq 0$. More efficient technologies correspond to larger parameters $A$ and $D$ in the aggregate production and environmental protection functions, and smaller parameter $B$ in the pollution function. Following Parente and Prescott (1994 and 2000), the institutional environment of a country influences the cost of adopting a new technology of production, environmental protection and pollution prevention. For example, lax environmental regulations decrease the private opportunity cost of using pollution intensive technologies, implying a larger parameter $B$ in the aggregate pollution function. Likewise, excessive bureaucracy increases the cost of environmental protection, making the parameter $D$ in the aggregate environmental protection function smaller. In what follows, we focus on Pareto optimality given the aggregate technological parameters $A$, $B$ and $D$ of the economy.

The effect of barriers to technology adoption will depend on the type of technological heterogeneity (in the production function, the environmental protection function or in the pollution function) and the specific steady state of the economy. Comparative statics on the steady state will give the direction of the effect of technological differences on the variables of in-
Table 1: Effect of Technology Adoption on $c$, $E$, $\pi$ and $K$

<table>
<thead>
<tr>
<th>Sign of Derivative</th>
<th>Linear $\Pi(\pi)$</th>
<th>Concave $\Pi(\pi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&gt; 0$</td>
<td>$\frac{dc}{dA}$, $\frac{d\pi}{dA}$, $\frac{dE}{dA}$, $\frac{dK}{dA}$, $\frac{dc}{dD}$, $\frac{dE}{dD}$, $\frac{dK}{dD}$</td>
<td>$\frac{dc}{dA}$, $\frac{d\pi}{dA}$, $\frac{dK}{dA}$, $\frac{dc}{dD}$, $\frac{dE}{dD}$, $\frac{dK}{dD}$</td>
</tr>
<tr>
<td>$\geq 0$</td>
<td>$\frac{d\pi}{dD}$, $\frac{dE}{dB}$</td>
<td>$\frac{dE}{dA}$, $\frac{d\pi}{dA}$, $\frac{dE}{dB}$</td>
</tr>
<tr>
<td>$&lt; 0$</td>
<td>$\frac{dc}{dB}$, $\frac{dE}{dB}$, $\frac{dK}{dB}$</td>
<td>$\frac{dc}{dB}$, $\frac{dK}{dB}$</td>
</tr>
</tbody>
</table>

Table 1 presents the resulting derivatives of the comparative statics for linear and strictly concave forms of the environmental protection function. The derivation of the more general results with $\Pi_{\pi\pi} \leq 0$ appears in appendix B.

From Table 1, we see that when environmental protection is linear, reducing barriers to technology adoption leading to improved aggregate efficiency will always improve environmental quality\(^1\). With respect to the environmental Kuznets curve, the middle row of the third column indicates that the possibility for a U-shaped curve for environmental quality will only exist for heterogeneity in TFPs ($A$) or pollution intensity of capital ($B$) when the environmental protection function $\Pi(\pi)$ is assumed to be strictly concave, thus exhibiting aggregate decreasing returns. To gain more insight into the conditions for an EKC as income varies, we focus on differences of the TFPs across countries (parameter $A$) due to its relative importance to national income (Parente and Prescott, 1994). The sign of the derivative of environmental quality with respect to the technical parameter $A$ will depend on the curvature of the utility and the environmental protection functions at each steady state, reflecting the rel-

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\(^1\)Improved technological efficiency in the pollution function $P(K)$ corresponds to lower values of the technical parameter $B$. 

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ative importance of capital and environmental quality in determining the optimal response of consumption and environmental expenditures to changes in TFPs. Proposition 2 summarizes this result.

**Proposition 2** The derivative of environmental quality with respect to the total factor productivity (TFP) depends on the curvature of the utility and environmental protection functions as follows:

\[
\frac{dE}{dA} < 0 \quad \text{if and only if} \quad \frac{\Pi_{\pi} d\pi}{\Pi_{\pi}} > \left( \frac{u_c}{u_c} - \frac{u_E}{u_E} \right) \frac{dc}{dA}.
\]

**Proof:** See appendix C.

For simplicity and in order to obtain further insight into the EKC, we specialize to constant elasticity utility and a simple strictly concave environmental protection function. The EKC will depend on the elasticity of intertemporal substitution of consumption, the degree of concavity of the environmental protection function, and the elasticities of consumption and environmental expenditures with respect to the total factor productivity (parameter \(A\)):

**Corollary 1** Define the utility function as \(u(c, E) = \varphi^{1-\sigma} + \psi^{1-\beta} + \frac{E^{1-\beta} - \frac{1}{1-\beta}}{1-\beta}, \) where \(\sigma, \beta, \varphi, \psi > 0,\) and the environmental protection function as \(D\Pi^{\varphi}(\pi) = D\pi^\delta, \) where \(0 < \delta < 1.\) Then,

\[
\frac{dE}{dA} < 0 \quad \text{if and only if} \quad \sigma \eta_c^A > (1 - \delta) \eta_{\pi}^A,
\]

where \(\eta_c^A = \frac{dc}{dA} \frac{A}{c} \) and \(\eta_{\pi}^A = \frac{d\pi}{dA} \frac{A}{\pi}\) are the elasticities of consumption and environmental expenditures with respect to the total factor productivity.

**Proof:** See appendix C.

From corollary 1, it follows that as \(\delta\) approaches 1, the term \((1 - \delta)\eta_{\pi}^A\) approaches zero. Consequently, since the term \(\sigma \eta_c^A = \sigma \frac{dc}{dA} \frac{A}{c}\) is strictly positive (Table 1), environmental quality
will increase with increases in productivity (parameter $A$). In the limiting case ($\delta = 1$), environmental protection is linear in environmental protection effort and environmental quality is always increasing with increases in $A$, as shown in Table 1.

The analytical results presented in Table 1 indicate that, with the exception of consumption and the stock of capital, the effect of technological improvements (larger $A$ or $D$, and smaller $B$) due to smaller institutional barriers is ambiguous. Therefore, there is potential insight to be obtained from numerical analysis of the comparative statics of the model. We perform comparative statics in the steady state of the dynamic model in the next section.

4 NUMERICAL ANALYSIS

This section reports numerical comparative statics of the steady state of the model with the following functional forms\(^2\):

$$u(c, E) = \alpha \ln(c) + (1 - \alpha) \ln(E),$$

$$B \cdot P^o(K) = BK^b,$$

$$D \cdot \Pi^o(\pi) = D\pi^\delta,$$

$$A \cdot F^o(K) = AK^m.$$

\(^2\)Notice that the utility function used here is obtained from $u(c, E) = \phi^{\frac{1-\sigma}{1-\sigma}} + \psi^{\frac{1-\beta}{1-\beta}}$ from Section 3.3 by letting $\sigma, \beta \to 1$, $0 < \phi = \alpha < 1$, and $\psi = (1 - \alpha)$. Maximizing the logarithmic case of the utility function is convenient since this is equivalent to maximizing the Cobb-Douglas case $u(c, E) = c^\alpha E^{1-\alpha}$. In the Cobb-Douglas formulation, $u_{cE} > 0$, reproducing the intuitive notion that higher environmental quality ($E$) makes consumption ($c$) more enjoyable.
With $0 < \alpha < 1$, $b \geq 1$, $0 \leq \delta \leq 1$, and $0 \leq m \leq 1$. More specifically, the baseline parameters are $\alpha = 0.8$, $b = 1.5$, $\delta = 0.15$, $m = 0.35$ and $\rho = 0.02$.

Due to the difficulty in obtaining aggregate data for the pollution function $B \cdot P(K)$ and the environmental protection function $D \cdot \Pi(\pi)$, no attempt was made to calibrate the model to real world circumstances. Instead, we use consolidated parameters in the literature when they are available and focus on the qualitative results of the numerical analysis. The value for the parameter $m$ corresponds to the share of capital in the production function in the U.S. of approximately $1/3$. The real discount factor $\rho = 0.02$ and the intertemporal elasticity of substitution $1/\sigma = 1$ are also typically used in the literature (see for example Cooley, 1995, and Barro and Sala-i-Martin, 1995). The choice of $\alpha$, $b$ and $\delta$ is arbitrary since little empirical information on these parameters is available. The value $\alpha = 0.8$ represents the relative importance of consumption compared to environmental quality. The values $b = 1.5$ and $\delta = 0.15$ produce a convex pollution function and concave environmental protection function respectively. Because of the importance of the parameters $\delta$ and $\sigma$ to the U-shaped curve for environmental quality as shown in corollary (1), other combinations are considered below. For ease of manipulation, the initial values of the technological parameters $A$, $B$ and $D$ are set equal to 10. Finally, the functional forms specified here conform to the assumptions in section 3.

Based on the preceding parameter values, a real-valued steady state solution is given by $c^* = 9.289$, $\pi^* = 0.335$, $K^* = 0.896$, and $E^* = 441.196$. We can verify that these values represent an equilibrium with saddle stability\textsuperscript{3}. The economic interpretation here is the usual one for equilibria presenting saddle stability. Since this is a deterministic model of the economy

\textsuperscript{3}To check for stability of the steady state, we linearize the system of differential equations describing optimality for $c$, $\pi$, $E$ and $K$ and calculate the eigenvalues of the resulting Jacobian matrix. Refer to Appendix D.
and we assume the social planner is fully rational, there is no reason to deviate from the optimal path to the steady state given the initial conditions of the state variables. We assume implicitly that the optimal path is feasible given the initial conditions\(^4\).

Figure 1 shows the response of consumption per capita, environmental protection effort, environmental quality and capital stock to differences in each of the technological parameters in the steady state. Recall that each point in Figure 4.1 corresponds to a different steady state obtained by a different set of parameters, in accordance with the use of comparative statics to describe different countries. Adoption of more efficient technologies corresponds to movement to the right along the horizontal axis. Thus, by construction, movement to the right along the horizontal axis corresponds to increasing \(A\) (total productivity of capital) and \(D\) (total environmental protection efficiency), and decreasing \(B\) (total pollution intensity of capital). The vertical axis measures the effect of adopting different technologies on the economic variables of interest. The curves labeled “dT” show the combined effect of simultaneous changes in \(A\), \(B\) and \(D\), whereas “dA”, “dB” and “dD” indicate the separate responses of the economic variables to more productive capital, less pollution intensive capital and more efficient preservation effort. The points where the lines cross on each of the graphs in Figure 1 are nothing but the baseline points for the numerical simulation, where \(A\), \(B\) and \(D\) all equal 10.

\(^4\)To guarantee feasibility, we can assume that the pristine level of the environment is large enough to assure the necessary initial stock of capital, and the resulting consumption and environmental preservation effort. That is, assume an extractive economy at time zero with enough resources to take the state and control variables to a stable path to the steady state. Clearly, this hypothesis is sufficient but not necessary to produce a feasible optimal path.
4.1 Effect of Technology Adoption on Economic Variables

We focus first on the effect of different TFPs (curves labeled “dA”) on the variables of the model. Higher values of the TFPs correspond to increasing consumption (figure 1(a)), environmental protection effort (figure 1(b)) and capital stock (figure 1(d)) in the steady state. More interestingly, the response of environmental quality to more productive capital is not monotone (figure 1(c)). Starting with small TFPs, marginal increases of this parameter cause environmental quality in the steady state to fall. The trend is eventually reversed and environmental quality rebounds. Thus, in a cross-section of countries, if we start with a country with a small TFP and income (large barriers to technology adoption) and compare it to another with a marginally larger TFP and income (marginally smaller barriers to technology adoption), the increased TFP will result in less environmental quality. This trend is eventually reversed as we look at richer countries with substantially larger TFPs (smaller barriers to technology adoption). This is consistent with Pareto optimality and the environmental Kuznets curve.

The effect of different TFPs on environmental quality follows from the system of equations (3) through (6), describing the steady state of the economy, together with the result in corollary 1. Equations (3) through (6) form a block recursive system. In particular, we can use equations (3), (5) and (6) to solve for $c^*$, $a^*$ and $K^*$ as functions of the parameters of the model. Then, equation (4),

$$ Nu_E \Pi_x - u_c \rho = 0, $$

provides necessary and sufficient conditions for the determination of environmental quality.
consistent with optimality. Therefore, from equation (6),

$$F(K) - Nc - \pi = 0,$$

as the parameter $A$ (TFP) approaches zero, so does production $F(K)$ and consequently consumption $c$ and preservation effort $\pi$. From corollary 1, for a sufficiently small value of parameter $\delta$ of the environmental protection function, marginal environmental protection ($\Pi_\pi$) goes to infinity faster than marginal utility ($u_c$). From equation (4), for optimality to result, the marginal utility of environmental quality has to be small, thus the high value of $E$. In other words, optimality requires that smaller consumption be offset by higher environmental quality. With sufficiently small consumption, the shadow value of capital is high relative to the shadow value of the environment. Therefore, smaller barriers to adoption of technologies that increase capital productivity favor an increase in the capital stock and a decrease of environmental quality. As we move to higher TFPs, however, production eventually increases to afford both more consumption and environmental protection, and environmental quality rebounds. This path is depicted by the curve “dA” in Figure 1(c).

As corollary 1 indicates, the shape of the curve for environmental quality depends crucially on the concavity of the environmental protection function. Figure 2 presents some alternative values of $\delta$ and $\sigma$ that are consistent with a U-shaped relationship of environmental quality to total factor productivity.

The numerical results in Figure 1 indicate that more efficient environmental protection
(curves labeled “$dD$”) corresponds to increasing consumption (figure 1(a)), environmental protection effort (figure 1(c)) and capital (figure 1(d)). The same is true for the adoption of technologies that make capital less pollution intensive (curves labeled “$dB$”), except for the response of environmental protection effort. Figure 1(b) shows how cleaner capital causes environmental protection effort to decline. The intuition behind this result is available from equation (5),

$$-P(K) + \Pi(\pi) = 0,$$

describing constant environmental quality in the steady state. Cleaner capital corresponds to smaller parameter $B$ and consequently less pollution $P(K)$ per unit of capital. As $B$ and $P(K)$ go to zero, equation (5) requires that preservation effort and aggregate environmental preservation also go to zero so as to keep environmental quality constant in the steady state. This explains the decrease in environmental protection effort in Figure 1(b).

The curves labeled “$dT$” show that the combined effect of smaller barriers to adoption of efficiency augmenting technology in aggregate production, aggregate pollution and aggregate environmental protection is to increase consumption (figure 1(a)), environmental protection effort$^5$ (figure 1(b)), environmental quality (figure 1(c)) and capital (figure 1(d)).

Under the conditions in corollary 1, Figure 1 indicates that smaller barriers to technology adoption promote increased consumption, environmental expenditures and stock of capital. The same is true for environmental quality, except for differences in TFPs (parameter $A$), which delineate a curve that is decreasing for larger barriers to technology adoption (lower TFPs) and increasing for smaller barriers to technology adoption (higher TFPs). This result

$^5$More precisely, environmental protection effort initially increases, but eventually decreases as $B$ alone falls to zero. This drop in environmental protection effort is not shown in Figure 1(b) to limit the scale of the vertical axis and allow meaningful comparisons of the curves of the graph.
is consistent with the cross-sectional evidence on the EKC reported in the literature.

5 CONCLUSION

This paper develops a dynamic model relating technology adoption and environmental quality. It shows how country-specific characteristics can help explain the existence of the environmental Kuznets curve. In particular, differences in total factor productivity can produce the U-shaped relationship of environmental quality and income depending on the shadow values of capital and environmental quality. An implication of this result is that institutional reforms that increase efficiency in production will not necessarily promote environmental quality gains. Furthermore, the traditional time series argument that the EKC is a byproduct of economic growth based on the assumption that countries are identical bears an extra burden of proof. Therefore, empirical research on environmental quality and economic development needs further consideration.

This paper also considered the effect of technologies that make capital less pollution intensive and environmental protection more effective. The results point to improved environmental quality and increased consumption and capital when these technologies are easily adopted. In the limiting case, highly clean capital enables a reduction in environmental protection expenditures.
A PROOF OF PROPOSITION 1

In the steady state rates of growth of the economic variables of the model are equal to zero. To see that, start with the equations for \((\gamma_c), (\gamma_\pi), (\gamma_K)\) and \((\gamma_E)\):

\[
\begin{align*}
\gamma_c & \equiv \frac{\dot{c}}{c} = -\frac{1}{\sigma} \left( \frac{P_K}{\Pi_\pi} - F_K + \rho \right) \\
\gamma_\pi & \equiv \frac{\dot{\pi}}{\pi} = \frac{1}{(\delta - 1)} \left( \gamma_c - \rho + N \frac{u_E}{u_c} \Pi_\pi \right) \\
\gamma_K & \equiv \frac{\dot{K}}{K} = \frac{F(K) - Nc - \pi}{K} \\
\gamma_E & \equiv \frac{\dot{E}}{E} = -\frac{P(K) + \Pi(\pi)}{E}
\end{align*}
\]

**Proposition 1** In the steady state the rates of growth of consumption \((\gamma_c)\), environmental expenditures \((\gamma_\pi)\), the stock of capital \((\gamma_K)\) and environmental quality \((\gamma_E)\) are equal to zero.

**Proof:** The result can be proven by way of contradiction. Start with the capital stock, and suppose \(\gamma_K > 0\). Then \(K \to \infty\) and consequently \(F_K \to 0\) and \(P_K \to \infty\). Since \(\gamma_c\) is also constant, the equation for \(\gamma_c\) implies that \(\pi \to 0\) so that \(\Pi_\pi \to \infty\). Now, \(K \to \infty\) and \(\pi \to 0\) imply that \(E \to 0\), not an optimal outcome, since \(\lim_{E \to 0} u_E = \infty\).

Suppose now that \(\gamma_K < 0\). Then \(K \to 0, F_K \to \infty\) and \(P_K \to \infty\). A constant \(\gamma_c\) requires that \(\pi \to \infty\), so that \(\Pi_\pi \to 0\). But \(K \to 0\) and \(\pi \to \infty\) is a contradiction to the feasibility condition that \(\pi = F(K) - Nc - \gamma_K K\).

Next, consider growth in consumption. Suppose \(\gamma_c > 0\). Then \(c \to \infty\) and that must result from ever increasing capital stock, i.e. \(K \to \infty\). But as in the case for \(\gamma_K > 0\), that yields a not optimal level of environmental quality. On the other hand, if \(\gamma_c < 0\), then \(c \to 0\) and that...
is also not optimal, since \( \lim_{c \to 0} u_c = \infty \).

Lastly, since the stock of capital and consumption are constant in the steady state \( (\gamma_K = \gamma_c = 0) \), then aggregate income or production \( (F(K)) \) is also constant and consequently so is environmental preservation effort and environmental quality. ■

### B COMPARATIVE STATICS

Total differentiation of (3)-(6) normalizing population so that \( N = 1 \) yields:

\[
\begin{pmatrix}
0 & -\frac{P_K}{\Pi^\pi} & \frac{P_{KK}}{\Pi^\pi} - F_{KK} & 0 & 0 & 0 \\
\Pi^\pi u_{Ec} - \rho u_{Ec} & \Pi^\pi u_{E} & \Pi^\pi u_{EE} - \rho u_{cE} & 0 & 0 & 0 \\
0 & \Pi^\pi & 0 & -P_K & 0 & 0 \\
-1 & -1 & 0 & F_K & 0 & 0
\end{pmatrix}
\begin{pmatrix}
dc \\
d\pi \\
dE \\
dK
\end{pmatrix}
\begin{pmatrix}
\frac{F^\pi_K dA}{\Pi^\pi} - \frac{P^\pi_K dB}{\Pi^\pi} + \frac{\Pi^\pi K dD}{\Pi^\pi} \\
0 dA + 0 dB - \Pi^\pi u_E dD \\
0 dA + P^\pi dB - \Pi^\pi dD \\
-F^\pi dA + 0 dB + 0 dD
\end{pmatrix}
\]

To simplify notation, rewrite the above as:

\[
\begin{pmatrix}
0 & x_1 & 0 & x_2 \\
x_3 & x_4 & x_5 & 0 \\
0 & \Pi^\pi & 0 & -P_K \\
-1 & -1 & 0 & F_K
\end{pmatrix}
\begin{pmatrix}
dc \\
da \\
dE \\
dK
\end{pmatrix}
\begin{pmatrix}
F^\pi_K dA - z_1 dB + z_2 dD \\
0 dA + 0 dB - z_3 dD \\
0 dA + P^\pi dB - \Pi^\pi dD \\
-F^\pi dA + 0 dB + 0 dD
\end{pmatrix}
\]
Let $\Lambda$ represent the first matrix on the left hand side. Then its determinant is given by

$$\det(\Lambda) = -x_5(x_1 P_K + x_2 \Pi_\pi) > 0$$

Apply Cramer’s rule to calculate the derivative of environmental quality with respect to changes in $A$ (changes in total factor productivity):

$$\frac{dE}{dA} = \frac{1}{\det(\Lambda)} \cdot \det \begin{pmatrix} 0 & x_1 & F_K^\circ & x_2 \\ x_3 & x_4 & 0 & 0 \\ 0 & \Pi_\pi & 0 & -P_K \\ -1 & -1 & -F^\circ & F_K \end{pmatrix}$$

$$= \frac{1}{\det(\Lambda)} \left[ -x_3 \det \begin{pmatrix} x_1 & F_K^\circ & x_2 \\ \Pi_\pi & 0 & -P_K \\ -1 & -F^\circ & F_K \end{pmatrix} + \det \begin{pmatrix} x_1 & F_K^\circ & x_2 \\ x_4 & 0 & 0 \\ \Pi_\pi & 0 & -P_K \end{pmatrix} \right]$$

$$= \frac{1}{\det(\Lambda)} \left[ -x_3 \left( F_K^\circ \Pi_\pi - x_2 F^\circ \Pi_\pi - x_1 F^\circ P_K - \Pi_\pi F_K F_K^\circ \right) + x_4 F_K^\circ P_K \right]$$

$$= \frac{1}{\det(\Lambda)} \left[ -x_3 \left( F_K^\circ \Pi_\pi \left( \frac{P_K}{\Pi_\pi} - F_K \right) - F^\circ (x_2 \Pi_\pi + x_1 P_K) \right) + x_4 F_K^\circ P_K \right]$$

$$= \frac{1}{\det(\Lambda)} \left[ -x_3 \left( -\rho F_K^\circ \Pi_\pi - F^\circ (x_2 \Pi_\pi + x_1 P_K) \right) + x_4 F_K^\circ P_K \right] \geq 0$$

Where the last equality follows from equation (3). Derivation of the remaining derivatives follows from the application of Cramer’s rule. The results follow:

$$\frac{dE}{dB} = \frac{1}{\det(\Lambda)} \left[ -x_3 \left( \rho z_1 \Pi_\pi + P^\circ (x_1 F_K + x_2) \right) + x_4 (x_2 P^\circ - z_1 P_K) \right] \geq 0$$
\[
\frac{dE}{dD} = \frac{1}{\det(\Lambda)} \left[ -x_3[-\rho_2 \Pi_\pi - \Pi^o(x_1 F_K + x_2)] + x_2(z_3 \Pi_\pi - x_4 \Pi^o) \right] > 0
\]

\[
\frac{dc}{dA} = \frac{1}{\det(\Lambda)} \left[ -x_5[\rho F_K^o \Pi_\pi + x_1 P_K F^o + x_2 F^o \Pi_\pi] \right] > 0
\]

\[
\frac{dc}{dB} = \frac{1}{\det(\Lambda)} \left[ x_5(\rho z_1 \Pi_\pi + x_2 P^o + x_1 F_K P^o) \right] < 0
\]

\[
\frac{dc}{dD} = \frac{1}{\det(\Lambda)} \left[ -x_5(\rho z_2 \Pi_\pi + x_2 \Pi^o + x_1 F_K \Pi^o) \right] > 0
\]

\[
\frac{d\pi}{dA} = \frac{1}{\det(\Lambda)} \left[ -x_5 F^o P_K \right] > 0
\]

\[
\frac{d\pi}{dB} = \frac{1}{\det(\Lambda)} \left[ x_5(z_1 P_K - x_2 P^o) \right] \geq 0
\]

\[
\frac{d\pi}{dD} = \frac{1}{\det(\Lambda)} \left[ -x_5(z_2 P_K - x_2 \Pi^o) \right] \geq 0
\]

\[
\frac{dK}{dA} = \frac{1}{\det(\Lambda)} \left[ -x_5 \Pi_\pi F_K^o \right] > 0
\]

\[
\frac{dK}{dB} = \frac{1}{\det(\Lambda)} \left[ x_5(x_1 P^o + z_1 \Pi_\pi) \right] < 0
\]

\[
\frac{dK}{dD} = \frac{1}{\det(\Lambda)} \left[ -x_5(x_1 \Pi^o + z_2 \Pi_\pi) \right] > 0
\]

C PROOFS OF PROPOSITION 2 AND COROLLARY 1

Proposition 2 The derivative of environmental quality with respect to the total factor productivity (TFP) depends on the curvature of the utility and environmental protection functions as follows: \( \frac{dE}{dA} \geq 0 \) if and only if \( \frac{\Pi_\pi}{\Pi^o} \frac{dx}{dA} \geq \left( \frac{u_c}{u_c} - \frac{u_E}{u_E} \right) \frac{dc}{dA} \).

Proof: To determine the condition for the sign of the effect of different TFPs on environmental quality, focus on the expression for \( dE/dA \) from appendix B substituting \( x_1, x_2, x_3 \) and \( x_4 \) with their corresponding expressions and using the normalization \( N = 1 \):
\[
\frac{dE}{dA} > 0 \iff 
\]
\[
\Pi_{\pi\pi}u_E F_K^\circ P_K > - (\Pi_{\pi} u_{E c} - \rho u_{cc}) \left\{ \rho F_K^\circ \Pi_{\pi} + F_K^\circ \left[ \left( \frac{P_{KK}}{\Pi_{\pi}} - F_{KK} \right) \Pi_{\pi} - \frac{\Pi_{\pi\pi}}{\Pi_{\pi}^2} P_K^2 \right] \right\}
\]
\[
\Pi_{\pi\pi} u_E > \Pi_{\pi} \left( \frac{\rho}{\Pi_{\pi}} u_{cc} - u_{Ec} \right) \left\{ \rho \frac{P_K}{F_K^\circ P_K} \left[ \left( \frac{P_{KK}}{\Pi_{\pi}} - F_{KK} \right) \Pi_{\pi} - \frac{\Pi_{\pi\pi}}{\Pi_{\pi}^2} P_K^2 \right] \right\}
\]

The term within braces on the right hand side of the expression above corresponds to the ratio of \(dc/dA\) to \(d\pi/dA\). Also make use of equation (4) to substitute \(u_E/u_c\) for \(\rho/\Pi_{\pi}\) and rewrite the above condition as follows:

\[
\frac{\Pi_{\pi\pi}}{\Pi_{\pi}} > \frac{1}{u_E} \left( \frac{u_E}{u_c} u_{cc} - u_{Ec} \right) \frac{dc/dA}{d\pi/dA}
\]
\[
\frac{\Pi_{\pi\pi} d\pi}{\Pi_{\pi} dA} > \left( \frac{u_{cc}}{u_c} - \frac{u_{Ec}}{u_E} \right) \frac{dc}{dA}
\]

**Alternative proof:** First notice that the system of equations describing the steady state is block recursive. Use equations (3), (5) and (6) to solve for \(c, \pi\) and \(K\) as functions of the model parameters. Next, apply the implicit function theorem to equation (4) to obtain:

\[
\frac{dE}{dA} = \frac{\Pi_{\pi\pi} \frac{d\pi}{dA} u_E + u_{E c} \Pi_{\pi} \frac{dc}{dA} - \rho u_{cc} \frac{dc}{dA}}{-\Pi_{\pi} u_{EE} + \rho u_{EE}} \geq 0
\]
\[
\iff \frac{d\pi}{dA} \frac{\Pi_{\pi\pi}}{\Pi_{\pi}} u_E \geq \left( \rho u_{cc} - u_{E c} \Pi_{\pi} \right) \frac{dc}{dA}
\]
⇔ \frac{\Pi_{\pi\pi}}{\Pi_{\pi}} d\pi \rho \geq \left( \rho u_{cc} - u_{E} \right) \frac{dc}{dA},

where the last expression was obtained by making use of equation (4). Rearranging:

\frac{dE}{dA} \geq 0 \iff \frac{\Pi_{\pi\pi}}{\Pi_{\pi}} d\pi \geq \left( \frac{u_{cc}}{u_{E}} \right) \frac{dc}{dA}.

\blacksquare

**Corollary 1** Define the utility function as 
\[ u(c_t, E_t) = \phi c_t^{1-\sigma} - \sigma c_t + \psi E_t^{1-\beta} - \beta E_t, \]
where \( \sigma, \beta, \phi, \psi > 0 \), and the environmental protection function as 
\[ D\Pi^o(\pi_t) = D\pi_t^\delta, \] where \( 0 < \delta < 1 \). Then,

\[ \frac{dE}{dA} \geq 0 \text{ if and only if } \sigma \eta^A_c \geq (1-\delta) \eta^A_{\pi}, \]

where \( \eta^A_c = \frac{dc}{dA} \) and \( \eta^A_{\pi} = \frac{d\pi}{dA} \) are the elasticities of consumption and environmental expenditures with respect to the total factor productivity.

**Proof:** For the utility and environmental protection functions defined above,

\[ \frac{\Pi_{\pi\pi}}{\Pi_{\pi}} = \frac{\delta - 1}{\pi}, \quad \frac{u_{cc}}{u_{c}} = -\frac{\sigma}{c} \quad \text{and} \quad u_{Ec} = 0. \]

Plugging these expressions into the result of proposition 2 yields:

\[ \frac{(\delta - 1) d\pi}{\pi dA} \geq -\frac{\sigma}{c} \frac{dc}{dA}. \]
Multiplying both sides by $A$ we obtain:

$$(\delta - 1) \frac{A}{\pi} \frac{d\pi}{dA} < - \sigma \frac{dc}{c} \frac{dA}{dA}.$$  

Rearranging:

$$\sigma \eta^A_c > (1 - \delta) \eta^A_\pi,$$

where $\eta^A_c = \frac{dc}{dA} \frac{A}{c}$ and $\eta^A_\pi = \frac{d\pi}{dA} \frac{A}{\pi}$ are the elasticities of consumption and environmental expenditures with respect to the total factor productivity.

\section*{D DYNAMIC SYSTEM STABILITY}

The conditions for optimality in our model are given by the system of differential equations (1), (2) and the equations of motion for $E$ and $K$:

\begin{align*}
\dot{c} &= u_{cc} \left( \frac{P_K}{\Pi_{\pi}} - F_K + \rho \right) - \frac{u_{cE} E}{u_{cc}} \dot{E}
\dot{\pi} &= \frac{\Pi_{\pi}}{u_{c} \Pi_{\pi \pi}} \left[ u_{cc} \dot{c} + u_{cE} \dot{E} + \Pi_{\pi} u_E - \rho u_c \right]
\dot{E} &= -P(K) + \Pi(\pi)
\dot{K} &= F(K) - Nc - \pi
\end{align*}

Linearization of the above system around the steady state yields:
Where, the first matrix on the right hand side is evaluated at the steady state values of the variables and $R^{(2)}$ is the remainder of the Taylor expansion involving derivatives of second order and higher. The remainder is assumed to be negligible in a sufficiently small neighborhood of the steady state. The steady state values of the variables are denoted here by “*”. If we define $\tilde{x} = (x - \bar{x})$, we can rewrite the above system as $\dot{\tilde{x}} = \Gamma \cdot \tilde{x}$, and stability of the above system will be given by the eigenvalues of $\Gamma$. For the steady state $c^* = 9.289$, $\pi^* = 0.335$, $K^* = 0.896$, and $E^* = 441.96$, given by the initial parameters, two positive and two negative eigenvalues result ($7.797, -7.780, 0.035$ and $-0.012$). This means that the system of differential equations describing optimality in the problem exhibits saddle path stability around the steady state.
E REFERENCES


T. M. Selden and D. Song. “Environmental Quality and Development: Is There a Kuznets


Figure 1: Technology Adoption, Consumption, Preservation Effort, Environmental Quality and Capital
Figure 2: Total Factor Productivity and Environmental Quality

Environmental Quality vs. Total Factor Productivity

- \( \sigma = 0.9; \delta = 0.15 \)
- \( \sigma = 1; \delta = 0.15 \)
- \( \sigma = 1; \delta = 0.30 \)
- \( \sigma = 1; \delta = 0.05 \)