The Environmental Kuznets Curve and Optimal Growth[1]

Ariaster B. Chimeli[2]
Columbia University

John B. Braden
University of Illinois at Urbana-Champaign

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[2]Mailing address: International Research Institute for Climate Prediction, 133 Monell Building, PO Box 1000, Palisades, NY 10964-8000, USA. E-mail: chimeli@iri.columbia.edu

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Abstract

Empirical studies suggest the existence of an environmental Kuznets curve: In the initial stages of economic development pollution increases, but eventually the trend is reversed and environmental quality rebounds. Previous efforts to model this phenomenon have relied upon the restrictive assumptions of intergenerational conflicts, ill-defined property rights or higher pollution intensity of more productive capital. This paper develops a simple neoclassical growth model that is free from these assumptions and provides a more general explanation of the evolution of economic growth and environmental quality based on the relative scarcity of capital. The model's predictions are consistent with the environmental Kuznets curve and two other empirical regularities: (i) during the initial stages of economic development, growth is high but there is negligible regulation and expenditures on environmental protection so that pollution levels rise; (ii) at later stages of economic development, better environmental quality is actively pursued, so that pollution is reduced, and economic growth rates are lower. We also show how a pollution tax or tradable permits can only implement the social optimal if accompanied by other taxes on consumption or profits. We solve for the time when environmental quality starts to improve and analyze its determinants. (JEL O13, Q20)

Keywords: Environmental Quality, Economic Growth, Environmental Expenditures, Pollution Tax, Tradable Pollution Permits.

The Environmental Kuznets Curve and Optimal Growth

Since the Brundtland Commission report (World Commission on Environment and Development, 1987), the potential to advance both economic growth and environmental quality has been a major focus of debate and analysis within and beyond the economics profession. Gene M. Grossman and Alan B. Krueger (1995) and Thomas M. Selden and Daqing Song (1994) placed the issue in sharp relief by documenting empirical relationships between national income levels and industrial pollutants. For several of these pollutants, Grossman and Krueger discovered a pattern that has become known as the environmental Kuznets curve (EKC). Cross-sectionally across countries, as income grows, both emissions and ambient concentrations first rise and then fall, generating an inverted U-shaped relationship – the EKC.

The EKC is a correlation in search of an underlying theoretical explanation. Because it was discovered in panel data strongly relying on their cross-sectional dimension, there is no *a priori* reason to expect that it describes the evolution of an individual economy over time or that particular policies can produce an EKC-type path. Furthermore, little is know about why different time paths are observed for different pollutants and why the data for other pollutants fail to fit the inverted U-shaped pattern.

In this paper, following the tradition of two-sector endogenous growth models with an initial imbalance in capital[1], we develop a simple dynamic model of economic growth and environmental quality[2]. Our analysis advances beyond previous work in three respects. First, we provide a simpler and more powerful explanation for the EKC than has previously been offered – an explanation based on the relative scarcity of different types of capital during a country's development. Second, we show that typical environmental policy instruments that implement the social optimum in a static framework, such as taxes and permits, by themselves are insufficient to produce the desired result in a dynamic model. In the dynamic economy described here, inducing the optimal level of pollution through pollution taxes and permits goes only part way toward the optimal provision of environmental quality. Third, our model is able to solve for the time when environmental protection starts to improve and investigate its determinants. In particular, we show how different efficiencies of expenditure

[1]See for example, Hirofumi Uzawa (1965), Robert E. Lucas (1988), Sergio Rebelo (1991) and Jordi Caballé and Manuel S. Santos (1993).

[2]James Andreoni and Arik Levinson (2001) and Alexander S.P. Pfaff, Shubham Chaudhuri and Howard L.M. Nye (2001a and 2001b) develop static models to explain the EKC. Static models tend to simplify the analysis, but may lead to policy recommendations that are not effective in a dynamic setup, as Nancy L. Stokey (1998) and this paper show.

cause some environmental quality problems to resist improvement longer than others.

In our model, individuals care about both the consumption of a private good and the stock of environmental quality, which is a public good. The advantage of modeling environmental quality in the utility function, as opposed to pollution, is in revealing that the optimal level of emissions is only part of the problem of optimally providing environmental quality. Creation of nature reserves, ecosystems rehabilitation, pollution sequestration, and species reintroduction are among the other means of environmental improvement. We introduce these options in stylized form and solve for the transitional dynamics and balanced growth paths of environmental quality, consumption, environmental protection expenditures, and productive capital. We show that the model can account for important empirical regularities in the relationship between growth and environmental quality.

Our model also is distinctive in representing environmental quality as a stock rather than a flow. Pollution degrades environmental quality, which can be restored only after the passage of some period of time – shorter for some aspects of environmental quality than others. For example, sulfur dioxide emissions are less important for their effects on ambient SO2 concentrations than for their enduring effects on soil chemistry, forest stocks, lake chemistry, and aquatic biodiversity (G. E. Likens et al., 1996 and C. Alewell et al., 2000). Persistent effects on environmental quality also result from chlorofluorcarbons (CFCs) (Mark Schrope, 2000) and many other industrial pollutants. Therefore, we focus on the state of the environment, characterized as a stock that can change as a result of emissions on one hand and corrective actions on the other. And, in contrast to much of the EKC literature, we are interested primarily in the relationship of growth and environmental quality – a public good – rather than to pollution. Since pollution diminishes environmental quality, in our framework, the EKC translates into an U-shaped relationship between income and environmental quality.

We are interested in the growth path that is optimal for an economy over time. We model this from the perspective of a social planner in an economy characterized by low initial income, abundant initial environmental quality, and the potential for endogenous growth. The transitional dynamics of the economy to an optimal balanced-growth path are consistent with three empirical regularities. First, environmental quality diminishes initially during a phase of intensive income growth, but eventually environmental quality improves as income growth continues, albeit at a slower rate. Thus, the optimal growth path for an individual economy is as implied by the EKC. Second, the planner will begin to invest in environmental improvement only after income and environmental quality reach threshold levels of capacity to invest in public goods and perceived need for environmental protection. At that point, investment in environmental improvement ratchets up quickly and continues thereafter to grow. This picture of benign neglect until reaching a threshold income level and stock of productive capital is consistent with the history of most of the developed countries. In the U.S., for example, most of the public commitment to environmental protection dates from the late 1960s and early 1970s, long after the introduction of regulations addressing other types of market failure (Paul E. Portney, 1990). Third, when environmental protection begins to be a focus of significant investment, income growth rates decrease. Some commentators have suggested that environmental protection is a drag on income growth (see for example, Edward F. Denison (1985), Dale W. Jorgenson and Peter J. Wilcoxen, 1990, and Nancy L. Stokey, 1998)[3].

Our analysis also sheds light onto the timing of reversal of environmental trends. For example, while acid precipitation has declined in North America and Europe (Kevin Krajick, 2001) and there is hope that stratospheric ozone may rebound with the international elimination of ozone-destroying compounds (Mark Schrope, 2000), global temperatures appear to be headed ever upwards as a result of ever-increasing greenhouse gas emissions (Douglas Holtz-Eakin and Thomas M. Selden, 1995, and Richard Schmalensee, Thomas M. Stoker, and Ruth A. Judson, 1998). Our model shows how variation in the efficiency of expenditures on prevention and remediation across different environmental problems can help to explain differences in rates of progress toward reversal.

Previous attempts to provide a theoretical explanation for the EKC owe much to the work of Bruce A. Forster (1973a,b) who introduced dynamic models of growth and environmental quality. Building on Forster's work, Thomas M. Selden and Daqing Song (1995) show the possibility of a "J" curve for abatement expenditures and an inverted "U" curve for pollution when the marginal utility of consumption is initially higher than the marginal benefit from abatement. However, Selden and Song indicate that this need not always occur, depending on the rate of growth of capital and consumption, and the response of pollution to abatement effort. By comparison, in this paper, we produce a definitive path for environmental quality and environmental expenditures consistent with the EKC. Furthermore, although Selden and

[3]By contrast, using measures of income that impute monetary value to health and amenities, Robert Repetto et al. (1996) calculate faster rates of growth in the presence of environmental protection. Likewise, other models that treat environmental quality as an input in production, such as A. Lans Bovenberg and Sjak A. Smulders (1996), indicate that growth may increase due to the tightening of environmental policy.

Song's analysis and this paper share a similar driving force for the EKC, we extend beyond their work by formally characterizing the transitional dynamics of the economy when environmental quality is initially abundant relative to capital, solving for the implementation of the social optimum in a decentralized economy, deriving explicitly the time when environmental quality starts to improve, and analyzing the difference in timing for the improvement of different environmental problems as a function of the efficiency of environmental protection.

More recently, Nancy L. Stokey (1998) derives the conditions for the existence of the EKC when more productive techniques of production are also more pollution intensive. In her analysis, at lower income levels, the marginal utility of consumption is high and pollution increases. If the marginal utility of consumption is not inelastic, then substitution of cleaner and less productive technologies for the dirtier and more productive ones causes pollution eventually to decrease. Stokey's analysis of the EKC is particularly impressive. She models different neoclassical and growth economies and also derives conditions for realizing an optimal growth-environment path in a decentralized economy. In a dynamic context, she derives the result that the price of a pollution permit equals the optimal tax on pollution for either a flow or a stock pollutant. She shows further that pollution standards cannot produce the optimal outcome.

Although the focus of Stokey's analysis is not on technological improvements in emissions, but rather substitution among techniques of production, her model implies a long-run migration towards the use of cleaner but less efficient techniques. Furthermore, along with Forster (1973b) and Selden and Song (1995), Stokey (1998) treats pollution as an increasing and convex function of capital. There is growing evidence, however, that capital efficiency increases and the pollution intensity of capital decreases in the long run as technology advances and market distortions are removed (Valerie Reppelin-Hill, 1999, and Madhu Khanna and David Zilberman, 1999). Thus, a fundamental driving force in the standard neoclassical approach to the EKC is open to doubt. Our model severs the link between capital and environmental quality by distinguishing pollution from environmental quality, where the latter can also be affected by public investments. We do not assume that capital productivity is positively associated with pollution intensity, and instead of pollution increasing at an increasing rate with the accumulation of productive capital, we focus on the possibility of a linear relationship that may nevertheless overstate the long-term prospects for pollution.

Our model differs from previous work not only in its characterization of the environment and of the technical relationship between productive capital and pollution, but also in admitting the possibility of public investment in environmental improvement apart from pollution reduction. As a result, we provide a more general result for the optimal set of public policies for growth and environment in a decentralized economy. Furthermore, our derivation of the EKC is not affected by the elasticity of marginal utility.

Several authors have advanced theoretical models of the EKC that emphasize the preferences of consumers or institutional factors. Andrew John and Rowena Pecchenino (1994) and Larry Jones and Rodolfo Manuelli (1994) focus on intergenerational conflicts, where the young strive for income while the elderly are interested in conservation. Graciela Chicilnisky (1994), Ramon Lopez (1994), and Henning Bohn and Robert T. Deacon (2000) focus on property rights and institutions. It is no doubt true that both preferences and institutions matter a great deal. But, our model abstracts from particular preferences or institutional structures to focus on relative scarcity – the most fundamental economic explanation of all – and we find that this is all that is required to produce the environmental Kuznets curve as an optimal path for a growing economy.

The paper proceeds as follows: Section I describes the model; section II investigates its steady state; section III analyzes the transition to the steady state, which is the essence of the EKC; section IV addresses policies for directing a decentralized economy along the optimal path; sections V and VI solve for the transition date and its response to the efficiency of environmental projection expenditures; section VII derives the optimal path of pollution abatement; section VIII explores a generalization of the utility function and shows how the EKC does not depend on the elasticity of marginal utility and is optimal even when we relax the assumption of balanced growth. Section IX draws conclusions.

I MODEL

Consider an economy modeled in continuous time t where each of the N identical individuals values consumption of a private good, c_t, and a pure public good, environmental quality, E_t. For simplicity, we assume that instantaneous individual utility is given by

$$u(c_t, E_t) = \alpha \ln(c_t) + (1-\alpha)\ln(E_t),$$

where the weight α on private consumption in utility is between zero and one. Individuals have a constant discount rate $0 < \rho < 1$.

Let K_t be the aggregate capital stock at time t. The consumption good is produced using the capital input according to a linear technology,

$$F(K_t) = AK_t,$$

where $A > 0$. Pollution, $P(K_t)$, is a by-product of production. For simplicity, we assume that pollution is a linear function of the capital employed in production,

$$P(K_t) = PK_t.$$

Pollution degrades the level of environmental quality in the economy. However, society can mitigate the effects of pollution on environmental quality by devoting resources, $\pi_t \geq 0$, to environmental protection. These environmental protection expenditures could include expenditures on pollution abatement, recovery of degraded areas, development of nature reserves, protection of endangered species, etc. Accordingly, environmental quality evolves over time according to

$$\dot{E}_t = -PK_t + \Pi\pi_t + \xi E_t,$$

where $\xi \geq 0$ allows for the natural regenerative capacity of the environment. Furthermore, ξ is assumed to be zero at the pristine state of the environment and positive otherwise. Perhaps more realistically natural recovery of the environment could be described by the function $g(E_t) \geq 0$ with $g'(E_t) \geq 0$, $\lim\limits_{E_t \to 0} g'(E_t) = 0$ and $\lim\limits_{E_t \to \bar{E}} g'(E_t) = 0$, where $\bar{E}$ represents the pristine state of the environment. However, this sophistication would introduce unnecessary complications and obscure the main intuition of the results.

It might be tempting to interpret $\Pi\pi_t$ as pollution abatement and the expression $-PK_t + \Pi\pi_t$ as net emissions at time t. This is not the case here, though. The variable π_t includes not only pollution abatement at time t, but also expenditures on recovery of degraded areas, development of nature reserves, protection of endangered species, etc. In a broader sense, environmental quality can be defined to include the built space such that, for example, there is an environmental improvement if a natural park becomes accessible to disabled citizens. This interpretation can be more appealing to long run balanced growth of the economy, if in fact it results from the decisions of the economic agents. In other words, environmental expenditures not only include abatement of the flow of pollutants at time t, but also recovery and improvement of the stock of the environment broadly defined. In that sense, the qualitative results remain unchanged if we replace π_t in the environmental protection function with $\pi_{1t}^{\beta}\pi_{2t}^{1-\beta}$, where π_{1t} are expenditures on pollution abatement, π_{2t} are other expenditures on environmental protection and $0 < \beta < 1$. In this formulation, each type of environmental protection expenditures exhibits diminishing returns, but constant returns to aggregate environmental expenditures are assumed for simplicity. The appendices contain the results with more than one type of environmental protection expenditures.

Finally, capital accumulation is the difference between production $F(K_t)$, aggregate consumption Nc_t, resources devoted to environmental protection π_t, and capital depreciation that occurs at the constant rate $0 < \delta < 1$:

$$\dot{K}_t = AK_t - Nc_t - \pi_t - \delta K_t.$$

We assume that the following transversality condition holds:

$$\lim_{t \to \infty} e^{-\rho t} \mu_t K_t = 0,$$

where μ_t is the current value of the time t shadow value of capital.

In this economy, we assume that a social planner seeking to maximize per capita lifetime utility chooses paths of consumption and environmental protection that solve:

$$\max_{c_t, \pi_t} \int_0^\infty e^{-\rho t} N[\alpha \ln(c_t) + (1-\alpha)\ln(E_t)]\, dt$$

subject to the laws of motion on environmental quality and capital accumulation

$$\dot{E}_t = -PK_t + \Pi\pi_t + \xi E_t,$$

$$\dot{K}_t = AK_t - Nc_t - \pi_t - \delta K_t,$$

$$\pi_t \geq 0,$$

and initial conditions

$$K_0, E_0.$$

The associated current value Lagrangian is given by

$$\mathcal{L}_t = N[\alpha \ln(c_t) + (1-\alpha)\ln(E_t)] + \lambda_t[-PK_t + \Pi\pi_t + \xi E_t] + \mu_t[AK_t - Nc_t - \pi_t - \delta K_t] + \theta_t \pi_t,$$

where μ_t is the shadow value of capital, λ_t is the shadow value of environmental quality, and $\theta_t \geq 0$ captures the non-negativity of environmental protection efforts.

The necessary conditions for a maximum are (see appendix A for the derivation of the necessary and transversality conditions):

$$\mu_t = \frac{\alpha}{c_t}, \tag{1}$$

$$\lambda_t = \frac{\mu_t - \theta_t}{\Pi}, \tag{2}$$

$$\dot{\lambda}_t = \lambda_t(\rho - \xi) - N\frac{(1-\alpha)}{E_t}, \tag{3}$$

$$\dot{\mu}_t = \mu_t \left(\frac{P}{\Pi} + \rho + \delta - A\right) - \frac{P\theta_t}{\Pi}, \tag{4}$$

$$\theta_t \pi_t = 0, \tag{5}$$

and the transversality condition:

$$\lim_{t\to\infty} e^{-\rho t} e^{\left(\frac{P}{\Pi}+\rho+\delta-A\right)t} \left(\tilde{\mu} - \frac{P}{\Pi}\int_0^t e^{-\left(\frac{P}{\Pi}+\rho+\delta-A\right)s}\theta_s ds\right) K_t = 0, \tag{6}$$

where $\tilde{\mu}$ is a constant.

At the optimum, equation (1) shows that the shadow value of the stock of capital equals the marginal utility of consumption, i.e. the marginal contribution of capital to social welfare must equal the marginal utility from additional consumption produced with the extra unit of capital. Equation (2) indicates the optimal trade-off between the stock of environmental quality and stock of capital, taking the marginal cost of improving environmental quality and the slackness condition into consideration.

Equations (3) and (4) form a system of differential equations with the laws of motion governing the shadow values of environmental quality and capital. At any given time, the rate of change in the shadow value of environmental quality is positively related to the discount rate and negatively related to the natural rate of recovery and marginal utility of environmental quality. Changes in the shadow value of the stock of capital, on the other hand, are positively related to the cost of marginal pollution in terms of environmental quality, the discount rate and the depreciation rate, and negatively related to the marginal product of capital.

II BALANCED GROWTH STEADY STATE

In a balanced growth steady state of the economy, every variable of the system grows at a common constant rate. Assuming an interior solution to the maximization problem above, the associated necessary conditions and the transversality condition are given in equations (1) through (6) with $\theta_t = 0$. The appendix provides a full derivation of the steady state results, which are summarized below:

The resulting optimal rate of growth of consumption is:

$$\frac{\dot{c}_t}{c_t} = \left(A - \frac{P}{\Pi} - \rho - \delta\right). \tag{7}$$

Assuming that the marginal product of capital is high enough to cover the marginal environmental protection cost per unit of capital, the discount rate and the capital depreciation rate, i.e. $\varphi \equiv \left(A - \frac{P}{\Pi} - \rho - \delta\right) > 0$, consumption will increase at the constant rate φ.

The necessary conditions also require a constant ratio between consumption and environmental quality for an optimal solution at each time t:

$$\frac{c_t}{E_t} = \left(A - \frac{P}{\Pi} - \delta - \xi\right) \frac{\alpha}{\Pi N(1-\alpha)},$$

or in a more compact notation $E_t = \phi c_t$, where ϕ is the inverse of the right hand side of the expression above. Also, by assumption, $\rho > \xi$. This assumption not only makes the fraction c_t/E_t positive, since $\varphi > 0$, but will prove useful below in the derivation of the transitional dynamics of the economy.

The results above, the initial conditions and the laws of motion for K and E, and the transversality condition imply the optimal path of the variables of the model over the planning horizon:

$$E_t = E_0 e^{\varphi t}, \tag{8}$$

$$c_t = \frac{E_0}{\phi} e^{\varphi t}, \tag{9}$$

$$K_t = \frac{(\varphi - \xi + \alpha\rho)}{(1-\alpha)\rho\Pi} E_0 e^{\varphi t}, \tag{10}$$

$$\pi_t = \left[\frac{(\varphi - \xi)}{\Pi} + \frac{P}{\Pi}\frac{(\varphi - \xi + \alpha\rho)}{(1-\alpha)\rho\Pi}\right] E_0 e^{\varphi t}. \tag{11}$$

Finally, using (8) and (10), a necessary condition for the optimal solution to hold in the steady state is that the ratio between the stocks of capital and environmental quality must be constant as follows:

$$\frac{K_t}{E_t} = \frac{(\varphi - \xi + \alpha\rho)}{(1-\alpha)\rho\Pi}. \tag{12}$$

Assuming that $\varphi > \xi$ yields a sufficient, but not necessary, condition for all the variables to be growing at the same rate γ^* in the steady state: If equation (12) holds, then a steady state with balanced growth results, $\gamma_c^* = \gamma_\pi^* = \gamma_K^* = \gamma_E^* = \gamma^* = \varphi$.

Clearly, in general, equality will not hold in equation (12) at time zero, and analysis of the transitional dynamics from the initial conditions to the steady state of the economy becomes a relevant exercise. This issue is addressed in the next section.

III TRANSITIONAL DYNAMICS

This section investigates the transitional dynamics of the economy described in the previous sections due to an imbalance on the initial conditions for capital and environmental quality. Typically, a country starts its process of economic development with a small stock of capital (i.e. small K_0) and a pristine environment (i.e. large E_0). In terms of equation (12), $\frac{K_0}{E_0} < \frac{(\varphi - \xi + \alpha\rho)}{(1-\alpha)\rho\Pi}$. A similar type of imbalance naturally emerges in endogenous growth models with physical and human capital, motivating their transitional dynamics[4].

The social planner wishes to get to the balanced endogenous growth path described in section II, therefore changing the initial ratio of capital to environmental quality so as to achieve equality in equation (12). One conceivable way to achieve this goal is to destroy part of the environmental stock and possibly convert that into capital so that equality in equation (12) is immediately achieved. However immediate conversion of the stock of the environment into capital is only possible to a limited extent in extractive societies, and it is more realistic to assume that these jumps are negligible and can be approximated by finite positive rates of investment in K_t and finite negative rates of growth of E_t. Negative growth in E_t will result from simultaneous increase in the capital stock and pollution, and zero expenditures on environmental protection, interpreted as investment in the stock of the environment.

Under the circumstances described above, the social planner recognizes the relative scarcity of K and its relatively high shadow value, thus increasing the stock of capital and allowing environmental quality to decrease. From equations (2) and (5), the relatively high shadow value of capital implies that the non-negativity constraint on environmental expenditures, π_t, is binding, and the term θ_t in the necessary conditions is positive. Furthermore, from equation

[4]For example, see Hirofumi Uzawa (1965), Robert E. Lucas (1988), Sergio Rebelo (1991) and Jordi Caballé and Manuel S. Santos (1993).

(4), the shadow value of capital decays faster when θ_t is positive:

$$\frac{\dot{\mu_t}}{\mu_t} = \left(\frac{P}{\Pi} + \rho + \delta - A\right) - \frac{P\theta_t}{\Pi\mu_t} < 0. \tag{13}$$

Equation (13) is consistent with higher rates of growth observed at earlier stages of development, when the capital stock of the economy is small relative to its environmental stock.

Manipulation of the necessary conditions produces the optimal consumption path during the transition to the steady state ratio of capital to environmental quality (see appendix A):

$$c_t = \frac{c_0 e^{\varphi t}}{1 - \frac{Pc_0}{\Pi\alpha}\int\limits_0^t \theta_s e^{\varphi s} ds}, \tag{14}$$

where c_0 is consumption per capita at time zero and θ_t is such that $\frac{Pc_0}{\Pi\alpha}\int\limits_0^t \theta_s e^{\varphi\tau} ds < 1$ for every t. Additionally, the equations of motion for capital ($\dot{K}_t = AK_t - Nc_t - \delta K_t$) and environmental quality ($\dot{E}_t = -PK_t + \xi E_t$) during the transition yield:

$$K_t = -Ne^{(A-\delta)t}\int\limits_0^t c_s e^{-(A-\delta)s} ds + K_0 e^{(A-\delta)t}, \tag{15}$$

$$E_t = -Pe^{\xi t}\int\limits_0^t K_s e^{-\xi s} ds + E_0 e^{\xi t}. \tag{16}$$

During the transition, when environmental quality is abundant relative to capital, the shadow value of capital, μ_t, will be decreasing by equation (13), implying that the stock of capital K_t will be increasing. To see that μ_t and K_t are inversely related, we solve for consumption in the equation of motion of capital, $c_t = \frac{(A-\delta)K_t - \dot{K}_t}{N}$, and substitute the right hand side into the necessary condition (1): $\mu_t = \frac{\alpha}{c_t}$. Rearranging and differentiating K_t with respect to μ_t yields: $\frac{\partial K_t}{\partial \mu_t} = -\frac{\alpha N}{\mu_t^2(A-\delta)} < 0$. Furthermore, we assumed that $\xi = 0$ at the pristine level of the natural environment, so environmental quality may initially stay close to the pristine level, but will eventually start to decrease as the capital stock increases. Subsequently, as environmental quality falls, E_t is concave in t, highlighting increasing environmental degradation as time elapses. The transition to the steady state is complete at the finite date t_1. At that time, the social planner faces a larger stock of capital than at time zero and consequently more pollution. To offset pollution and promote environmental improvements as described by the optimal steady state solution, a strictly positive level of environmental expenditures π_{t_1} is required. Furthermore, consumption begins to grow at a reduced rate. Proposition 1 summarizes these results.

Definition 1 *Environmental quality is abundant at time t if and only if the ratio of capital to environmental quality is smaller than the optimal steady state ratio, i.e.* $\frac{K_t}{E_t} < \frac{(\varphi-\xi+\alpha\rho)}{(1-\alpha)\rho\Pi}$.

Proposition 1 *For a country starting its planning horizon with abundant environmental quality:*

(i) for a sufficiently large initial stock of capital K_0, environmental quality is decreasing and concave in time during the transition to the steady state. Furthermore, the transition takes a finite period of time — there is a finite time t_1 such that environmental quality is increasing at the constant rate φ for every $t \geq t_1$;

(ii) environmental expenditures π_t are: (a) zero from time zero to t_1, and then (b) increasing at the constant rate φ at dates $t \geq t_1$. Furthermore, π_t is discontinuous at t_1;

(iii) the rate of growth of consumption from time zero to t_1, γ_c, exceeds its (constant) rate of growth φ at dates $t \geq t_1$.

Proof:

(i) From conditions (2) and (5), at t_0, the relatively high (low) shadow value of capital (environmental quality) requires zero expenditures on environmental protection π_t. Hence, the equation of motion of environmental quality becomes $\dot{E} = -PK + \xi E$. Choose K_0 large enough so that $\dot{E} < 0$. Then, $\ddot{E} = -P\dot{K} + \xi\dot{E} < 0$, since $\dot{K} > 0$ during the transition. Therefore, environmental quality E_t is decreasing and concave in time during the transition to the steady state. To see that the transition to the steady state is done in finite time, notice that since environmental quality is decreasing and capital is increasing, $E_t \to 0$, $K_t \to \infty$ and the ratio $\frac{K_t}{E_t} \to \infty$ as $t \to \infty$. Thus, starting with abundant environmental quality at t_0, there exists a $t_1 < \infty$ such that $\frac{K_{t_1}}{E_{t_1}} = \frac{(\varphi-\xi+\alpha\rho)}{(1-\alpha)\rho\Pi}$. That is, at time t_1 the optimal steady state ratio of capital to environmental quality is reached and the solution to the social planner's problem derived in section II indicates that environmental quality grows at the constant rate φ.

(ii) Zero expenditures on environmental protection when environmental quality is abundant follows directly from abundant environmental quality as defined above and conditions (2) and (5). Likewise, at time t_1 defined in (i), the steady state solution to the social planner's problem indicates that environmental expenditures, π_t, grow at the constant rate φ. To see that π_t is discontinuous at t_1, notice that $\pi_t = 0$ for $0 \leq t < t_1$ and $\pi_t = \left[\frac{(\varphi-\xi)}{\Pi} + \frac{P}{\Pi}\frac{(\varphi-\xi+\alpha\rho)}{(1-\alpha)\rho\Pi}\right] E_0 e^{\varphi t}$

Figure 1: [Approximately Here.]

for $t \geq t_1$. Therefore,

$$\lim_{t \to t_1^-} \pi_t = 0,$$

and

$$\lim_{t \to t_1^+} \pi_t = \left[\frac{(\varphi - \xi)}{\Pi} + \frac{P}{\Pi} \frac{(\varphi - \xi + \alpha\rho)}{(1-\alpha)\rho\Pi} \right] E_0 e^{\varphi t_1} > 0,$$

by assumption that the term within brackets in the last expression is strictly positive, since $\pi_t \geq 0$.

(iii) Define $\gamma_c^* \equiv \frac{\dot{c}}{c} = \varphi$ as the rate of growth of consumption in the steady state, i.e. for every $t \geq t_1$, with t_1 defined in (i). Next differentiate the necessary condition (1) with respect to time to obtain $\dot{\mu}_t = -\frac{\alpha \dot{c}_t}{c_t^2}$. Using (1) and rearranging yields $\gamma_\mu \equiv \frac{\dot{\mu}}{\mu} = -\frac{\dot{c}}{c} \equiv -\gamma_c$. Hence, from equation (13), during the transition, $\gamma_c = \left(A - \frac{P}{\Pi} - \rho - \delta \right) + \frac{P\theta_t}{\Pi\mu_t} = \varphi + \frac{P\theta_t}{\Pi\alpha} c_t > \varphi = \gamma_c^*$. ■

In accordance with Proposition 1 (i) and (ii), Figure 1 depicts the shape of the Pareto optimal curves for environmental quality and environmental expenditures as functions of time when a country starts its planning horizon with abundant environmental quality. These paths are consistent with the environmental Kuznets curve and the delayed environmental expenditures in most countries.

In Figure 1, environmental quality decreases during a transition phase. It is concave in time during the transition reflecting the effect of an increasing stock of productive capital accompanied by zero environmental expenditures. When the optimal ratio of capital to the environment is reached, environmental expenditures become positive and environmental quality starts to improve. From this date on, environmental quality, environmental expenditures, capital and consumption all grow at a common constant rate.

IV DECENTRALIZED ECONOMY

In this section, we analyze the problem of implementing the Pareto optimal solution in a dynamic decentralized economy. In particular we focus on a pollution tax, tradable permits for pollution, a pollution standard and a consumption tax. From proposition 1, when environmental quality is abundant, expenditures on environmental protection are zero. The approach to the optimal ratio of capital to environmental quality without government intervention is Pareto optimal (see section V). Hence, the analysis in this section focuses on the implementation of the first best along the balanced growth path of the economy following the point of transition.

The typical economic instruments to correct pollution externality problems are a pollution tax or tradable pollution permits. One of these instruments will suffice to implement the social optimum in a static version of the problem considered here, but not in its dynamic setup. In the dynamic context, imposing a pollution tax or introducing tradable pollution permits is only part of the implementation of the social planner's optimum. A pollution tax (or equivalently a tax on capital, since pollution is linearly proportional to the capital stock) leads consumption, capital and pollution to grow at the optimal rate (although at least one of them at the wrong level), but will only cause environmental quality to grow at its natural rate of recovery ξ, which is different from the optimal rate of growth φ. In other words, a single tax on pollution or capital will generate revenues that are enough to neutralize the effect of the pollution flow at time t, but it will not generate enough revenues for the optimal provision of the public good environmental quality: neither will enough environmental recovery be promoted, nor will other improvements as in the social planner's solution. The optimal path for the economy cannot be attained even if cumulative pollution at time t is taxed. Appendix B contains the derivations of these results. The optimum will require a tax on pollution and a tax on consumption in order to generate enough revenues for the provision of environmental quality beyond pollution control promoted by a pollution tax alone. With freely distributed pollution permits, a tax on profits has to be imposed, as well as a tax on consumption. We turn to these two cases now.

To implement the social planner's optimum with a tax on pollution (or equivalently on capital), the government has to simultaneously impose a tax on consumption. In the decentralized economy, the representative households hold capital, which they rent to firms and whose returns are used for consumption, payment of consumption taxes and savings. They maximize the stream of the overall utility from time zero to infinity, subject to changes in their stock of capital due to savings. Formally, their problem is:

$$\max_{c_t} \int_0^\infty e^{-\rho t} N[\alpha \ln(c_t) + (1-\alpha)\ln(E_t)]\, dt$$

subject to

$$\dot{K}_t = rK_t - N(1+\tau_c)c_t,$$

and the transversality condition

$$\lim_{t\to\infty} e^{-\rho t}\nu_t K_t = 0,$$

where r is the rental rate of capital, τ_c is the tax on consumption, and ν_t is the current value shadow value of capital.

The current value Hamiltonian for the households' problem is:

$$\mathcal{H} = N[\alpha \ln(c_t) + (1-\alpha)\ln(E_t)] + \nu_t[rK_t - N(1+\tau_c)c_t],$$

and the first order conditions imply:

$$\frac{\dot{\nu}_t}{\nu_t} = \rho - r,$$

and we use the conjecture to be verified later that the consumption tax τ_c is time invariant to obtain

$$\frac{\dot{c}_t}{c_t} = r - \rho.$$

The firms' problem is static and since the technology exhibits constant returns to scale, the industry can be represented by a single firm. At each time t, each firm faces a tax on pollution, τ_p, and chooses capital to maximize profits:

$$\max_{K_t} AK_t - rK_t - \delta K_t - \tau_p P K_t.$$

The first order condition gives the competitive price of capital:

$$r = A - \delta - \tau_p P.$$

The optimum derived from the social planner's problem is implemented in the decentralized economy if and only if the rate of growth of consumption is identical in both problems and the first order condition for the firm is satisfied. This implies that the tax on pollution must equal the inverse of marginal environmental protection or the efficiency of environmental protection expenditures:

$$\tau_p = \frac{1}{\Pi}.$$

That is the pollution tax equals the units of output spent on an additional unit change of environmental quality per unit of time.

Thus, the rental price of capital along the balanced growth path is constant and given by

$$r = A - \delta - \frac{P}{\Pi} = \varphi + \rho,$$

and the consumption path, after choosing the initial consumption level according to the social planner's solution, is

$$c_t = \frac{E_0}{\phi} e^{\varphi t}.$$

To obtain the consumption tax τ_c that implements the economy's first best, we solve for the stock of capital according to the equation of motion for capital:

$$\dot{K}_t = rK_t - N(1+\tau_c)c_t = (\varphi + \rho)K_t - N(1+\tau_c)\frac{E_0}{\phi}e^{\varphi t}.$$

By using the tranversality condition the stock of capital at time t is given by

$$K_t = \left(\frac{N}{\phi} + \frac{N\tau_c}{\phi}\right)\frac{E_0}{\rho}e^{\varphi t},$$

implying that the optimal tax on consumption that makes capital identical to the social planner's optimum capital stock in equation (10) is

$$\tau_c = \frac{(1-\alpha)}{\alpha}\frac{(\varphi - \xi)}{(\varphi + \rho - \xi)}.$$

Tax revenues to be used in the provision of the public good environmental quality are given by tax revenues from consumption plus tax revenues from pollution:

$$TR_t = \tau_c c_t N + \tau_p P K_t = \frac{(\varphi - \xi)}{\Pi}E_0 e^{\varphi t} + \frac{P}{\Pi}K_t = \pi_t,$$

as in the first best level of expenditures on environmental protection in equation (11). Clearly, our conjecture of a time invariant τ_c works here and environmental quality at time t will follow the same path as in the social planner's solution.

Next consider tradable pollution permits freely distributed by the government. Freely distributed tradable pollution permits are designed to promote the socially optimal level of emissions by firms at each time t, not to generate tax revenues. To obtain the social planner's solution the government has to tax consumption and profits that may result from the distribution of the permits so as to generate enough tax revenues for the optimal provision of environmental quality.

With this new set of instruments, the households' problem becomes

$$\max_{c_t} \int_0^\infty e^{-\rho t} N[\alpha \ln(c_t) + (1-\alpha)\ln(E_t)]\, dt$$

subject to

$$\dot{K}_t = rK_t - N(1+\tau_c)c_t + (1-\tau_\eta)\eta_t,$$

where η_t are any profits that the firm generates to households and τ_η is the tax rate on profits. As before, the same transversality condition applies. As in the tax case, the first order conditions for the households' problem implies

$$\frac{\dot{c}_t}{c_t} = r - \rho.$$

The firm's problem is now

$$\max_{K_t} AK_t - rK_t - \delta K_t - p(PK_t - PK_t^*),$$

where p is the price of the pollution permit and PK_t^* is the quantity of permits distributed by the government at time t. The firm's problems under the permit and tax schemes are quite similar. The rental price of capital is

$$r = A - \delta - pP,$$

implying that profits are equal to the value of the permits distributed to the firm:

$$\eta_t = pPK_t^*.$$

The permit price that implements the optimal solution in a competitive equilibrium is identical to the pollution tax previously analyzed:

$$p = \frac{1}{\Pi}.$$

Consequently, the rental price of capital is the same as before

$$r = A - \delta - \frac{P}{\Pi} = \varphi + \rho,$$

and consumption grows at the optimal rate and level with the appropriate initial condition:

$$c_t = \frac{E_0}{\phi} e^{\varphi t}.$$

The stock of capital at time t is the solution to the law of motion of capital

$$\dot{K}_t = rK_t - N(1+\tau_c)c_t + (1-\tau_\eta)\eta_t = (\varphi+\rho)K_t - N(1-\tau_c)\frac{E_0}{\phi}e^{\varphi t} + (1-\tau_\eta)\eta_t,$$

and the optimum can be clearly implemented if the tax on consumption is the same as before and the tax rate on profits is equal to 1, i.e.,

$$\tau_\eta = 1.$$

This way, the equation for capital becomes identical as in the tax scheme, so that $K_t = K_t^*$, and tax revenues are

$$TR_t = \tau_c c_t N + \tau_\eta \eta_t = \frac{(\varphi - \xi)}{\Pi} E_0 e^{\varphi t} + \frac{P}{\Pi} K_t = \pi_t.$$

In comparison with the pollution tax case, freely distributed pollution permits transfer revenues from the government to firms in the form of profits by the same amount as tax revenues from pollution. To implement the social optimum, the government needs to recover these revenues, and thus has to tax profits by 100%.

If instead of distributing the pollution permits the government decides to sell them to firms at the price p, their problem becomes

$$\max_{K_t} AK_t - rK_t - \delta K_t - pPK_t,$$

which is identical to the pollution tax problem. Thus, the optimum can be implemented with the sales of permits and a consumption tax.

Finally, suppose the government imposes a pollution standard, thus imposing a limit on the stock of capital to be used at each time t, and sets a consumption tax as before[5]. The households' problem remains the same and the firm's zero-profit condition implies that $r = A - \delta$. This rental price is different from the one in the first best solution and the Pareto optimal paths for the variables of the model cannot be attained. The intuition for the failure of a pollution standard to implement the optimum in a dynamic economy is well explained by Nancy L. Stokey (1998, p.24):

"With a pollution tax ..., emissions have a market price that is entirely separate from the return to capital. Hence the market return to capital is an accurate measure of the incremental value of investment and provides the correct incentive to save. With direct regulation, ownership of capital brings with it the right to emit a specified level of pollution. Thus, the

[5]Setting a pollution standard without a consumption tax does not impact the qualitative result that the first best solution cannot be attained.

market return to capital is the sum of its real economic return and the value of the associated pollution rights. The bundled price overstates the return to investment and provides the wrong incentives for accumulation."

The extension of the results from this section to the case of more than one type of environmental protection expenditures, each exhibiting diminishing returns, but with aggregate constant returns follows easily. Appendix B contains the corresponding derivations for the environmental protection function represented by $\Pi\pi_{1t}^{\beta}\pi_{2t}^{1-\beta}$.

V TRANSITION TIME

In this section we compute the time when environmental quality starts to improve. This way, we can investigate the determinants of the timing of environmental improvements and shed light on why some environmental problems tend to persist longer than others. The computation of the transition time relies on the policy of no government intervention during the approach to the optimal ratio of capital to environmental quality. Thus, we begin with the result such policy is Pareto optimal.

Proposition 2 *Starting with abundant environmental quality, the approach to the optimal ratio of capital to environmental quality without government intervention (no taxes) is Pareto optimal.*

Proof: See Appendix C.

Proposition 2, implies that consumption per capita, capital and environmental quality are given as follows (see Appendix C):

$$c_t = \frac{\rho K_0}{N} e^{(A-\delta-\rho)t},$$
$$K_t = K_0 e^{(A-\delta-\rho)t},$$

and

$$E_t = -\frac{P}{(A-\delta-\rho-\xi)} K_0 e^{(A-\delta-\rho)t} + \left[E_0 + \frac{PK_0}{(A-\delta-\rho-\xi)}\right] e^{\xi t}.$$

Furthermore the ratio of capital to environmental quality during the transition is:

$$\frac{K_t}{E_t} = \frac{K_0 e^{(A-\delta-\rho)t}}{-\frac{P}{(A-\delta-\rho-\xi)} K_0 e^{(A-\delta-\rho)t} + \left[E_0 + \frac{PK_0}{(A-\delta-\rho-\xi)}\right] e^{\xi t}}. \tag{17}$$

According to the notation in proposition 1, let t_1 denote the date when environmental quality begins to improve. At t_1 the ratio of capital to environmental quality in the unregulated economy is the same as the optimal ratio in equation (12). Setting (17) equal to (12), we can calculate t_1:

$$t_1 = -\frac{1}{(A-\delta-\rho-\xi)} \ln \left[\frac{K_0(A-\delta-\rho-\xi) + K_0 PR}{E_0(A-\delta-\rho-\xi)R + K_0 PR}\right] > 0,$$

where R is the optimal ratio of capital to environmental quality along the balanced growth path derived in equation (12),

$$R = \frac{\varphi - \xi + \alpha\rho}{(1-\alpha)\rho\Pi},$$

and $t_1 > 0$ follows from the initial condition that $\frac{K_0}{E_0} < R$. Appendix C contains the derivations.

With the exception of population size, all the parameters of the model plus the stock of capital and the environment at time zero affect the transition time to balanced growth in the unregulated economy. A higher weight of consumption, α, in the utility function delays the transition time by increasing the optimal ratio of capital to environmental quality. A larger initial stock of the environment causes the duration of the transition to increase, whereas a larger initial stock of capital has the opposite effect. Higher pollution intensity of capital, P, causes the optimal ratio of capital to environmental quality to decrease and the transition time to accelerate.

The effects of the other parameters of the model on the transition time are ambiguous. In particular, increases in the depreciation rate, δ, and the discount rate, ρ, contribute to a smaller optimal ratio of capital to environmental quality, thus reducing the transition time, but also cause the rate of growth of capital and rate of decay of environmental quality to decrease during the unregulated transition, thus increasing the transition time. A larger total factor productivity, A, on one hand delays t_1 through a larger optimal ratio of capital to the environment, but on the other hand, it accelerates t_1 through a faster rate of growth of capital and decay of environmental quality during the transition.

Although we cannot determine the effect of the efficiency of environmental protection expenditures, Π, on the transition time *a priori*, we can show that the behavior of t_1 in response to different values of Π mimics the behavior of the optimal ratio of capital to environmental quality, R, in response to different values of Π. This fact sheds light on the reason why different environmental problems can have different turning points or transition times, all else held constant. The next section focuses on response of the transition time to different efficiencies of environmental protection expenditures.

VI ENVIRONMENTAL PROTECTION EFFICIENCY AND THE TRANSITION TIME

The preceding analysis implicitly assumes that expenditures on environmental protection can effectively improve environmental quality. In other words, the term Π – efficiency of total expenditures on environmental protection – is positive. Nonetheless, the efficiency of environmental protection expenditures will differ across environmental problems, thereby affecting the optimal ratio of capital to environmental quality and the time of transition to balanced growth.

For example, because it is so difficult to detect and monitor, nonpoint source pollution tends to be very costly to observe and control. Attacking global warming is difficult because it is so tightly connected to entrenched and pervasive energy technologies based on fossil fuels, and replacement technologies are not yet inexpensive or available enough to meet the rapidly growing demand for power. At the extreme, the cost of reintroducing an extinct species is infinity and the efficiency of one dollar spent on such an attempt is zero. Institutional factors may also affect the efficiency of environmental protection expenditures in a given economy, as in the case of greenhouse gases such as CO_2, CO and methane, where the environmental problem is inherently global and a solution involves global coordination. These characteristics of different environmental problems translate into different efficiencies of environmental protection expenditures and different turning points for environmental quality. Not surprisingly, empirical studies of some pollutants do not show any evidence of an environmental Kuznets curve. In this section, we analyze the effect of different efficiencies of environmental protection on the optimal ratio of capital to environmental quality and the time when environmental quality starts to improve in a decentralized economy.

To see the effect of the efficiency on environmental protection on the transition time, we first focus on its effect on the optimal ratio of capital to environmental quality. As before, let R denote the optimal ratio of capital to environmental quality as in equation (12), and assume that Π is such that $R \geq 0$. That is, set the numerator of equation (12) greater than or equal to zero and solve for Π:

$$\varphi - \xi + \alpha\rho = A - \frac{P}{\Pi} - \delta - \rho - \xi - \alpha\rho \geq 0 \therefore$$

$$\Pi \geq \frac{P}{A - \delta - \xi - (1-\alpha)\rho}. \tag{18}$$

Next, we solve for the derivative of R with respect to Π:

$$\frac{\partial R}{\partial \Pi} = \frac{2\frac{P}{\Pi} - A + \delta + \xi + (1-\alpha)\rho}{(1-\alpha)\rho\Pi} \gtrless 0 \quad \text{iff} \quad \Pi \lessgtr \frac{2P}{A - \delta - \xi - (1-\alpha)\rho}. \tag{19}$$

If $A - \delta - \xi - (1-\alpha)\rho < 0$, then any $\Pi \geq 0$ implies that $\frac{\partial R}{\partial \Pi} < 0$, i.e, environmental problems for which the efficiency of a dollar of environmental protection is smaller are associated with a higher optimal ratio of capital to environmental quality.

On the other hand, if $A - \delta - \xi - (1-\alpha)\rho > 0$, the sign of the derivative of R with respect to Π is ambiguous. For small enough Π, it is positive, and for large enough Π it is negative. It is also easy to verify that $\lim_{\Pi \to 0} R = -\infty$, $\lim_{\Pi \to \infty} R = 0^+$, so R has a unique positive maximum – this follows from (19). Additionally, $\Pi|_{R=0} < \Pi|_{R\text{max}}$ – this follows from (18) and (19). For sufficiently small efficiency of environmental protection, Π, the damage from capital relative to the efficiency of a dollar of environmental protection is high and it is optimal to have a small ratio of capital to environmental quality along the balanced growth path. For higher efficiency of environmental protection it pays to have a higher stock of capital relative to environmental quality, until Π is so high that it is optimal to take advantage of the environmental quality returns per dollar spent on environmental protection, and decrease the stock of capital relative to environmental quality.

The behavior of the transition time t_1 in response to different efficiencies of environmental protection mimics the behavior of the optimal ratio of capital to environmental quality:

$$\frac{\partial t_1}{\partial \Pi} = \left[\frac{1}{(A - \delta - \rho - \xi + PR)R}\right] \frac{\partial R}{\partial \Pi}.$$

Since the term in brackets is positive, by assumption, the derivative of t_1 with respect to Π has the same sign as the derivative of R with respect to Π.

Figures 2(a) and (b) plot R and t_1 as functions of a continuum of efficiencies of environmental protection for different environmental problems, given the same initial stock of capital and the environment. The initial ratio of K to E is indicated by the dashed line in figure 2(a), whereas the solid curve indicates the optimal ratio of K to E. For Π beyond the extremes of

Figure 2: [Approximately Here.]

the curves plotted in figure 2(a), the imbalance between capital and environmental quality goes in the opposite direction of definition 1 and is beyond the scope of this paper[6]. Figure 2(b) plots the corresponding transition times or turning points for environmental quality. For environmental problems associated with a small enough Π, marginal increments in Π imply a longer time until environmental quality starts to improve. Likewise, for environmental problems with a large enough Π, more efficient environmental protection implies a shorter time until environmental quality starts to improve.

VII POLLUTION ABATED BY FIRMS

The implementation mechanisms analyzed in section IV causes firms to reduce the stock of capital and consequently pollution to the optimal level. In particular, pollution abated by firms along the balanced growth path is equal to the difference between pollution in an unregulated economy and pollution that prevails under government intervention. From section V, we have the equation for capital in the unregulated economy

$$K_t^U = K_0 e^{(A-\delta-\rho)t},$$

and from section IV the implied equation for capital in the regulated economy starting at time t_1 is

$$K_t^R = \frac{(\varphi - \xi + \alpha\rho)}{(1-\alpha)\rho\Pi} E_{t_1} e^{\varphi t} = R E_{t_1} e^{\varphi t}, \tag{20}$$

where R is the optimal ratio of capital to environmental quality defined as before and $t > t_1$. Since R is achieved at t_1, we can rewrite (20) as

$$K_t^R = \frac{K_{t_1}}{E_{t_1}} E_{t_1} e^{\varphi t} = K_{t_1} e^{\varphi t} = K_0 e^{(A-\delta-\rho)t_1} e^{\varphi t}.$$

Hence, pollution abated by firms for $t > t_1$ is equal to

$$PK_t^U - PK_t^R = PK_0 e^{(A-\delta-\rho)t} - PK_0 e^{(A-\delta-\rho)t_1} e^{\varphi(t-t_1)} =$$

$$PK_0 e^{(A-\delta-\rho)t}(1 - e^{-\frac{P}{\Pi}(t-t_1)}).$$

[6]For an analysis of the imbalance given by scarce environmental quality at time zero, see XXXX(XXXX).

Clearly, the optimal path for environmental quality implemented in section IV implies that the government engages further pollution abatement, and environmental recovery and improvement, thus optimally providing the public good environmental quality.

VIII CONSTANT ELASTICITY OF SUBSTITUTION UTILITY

In this section we consider a more general utility function with constant elasticity of intertemporal substitution, given by

$$u(c_t, E_t) = \Psi \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \Lambda \frac{E_t^{1-\eta} - 1}{1-\eta},$$

where Ψ, Λ, σ, and η are positive constants. The preceding analysis refers to the case when $\Psi = \alpha$, $\Lambda = 1-\alpha$, and $\sigma, \eta \to 1$.

To avoid unnecessary complications, we only focus on the linear environmental protection function $\Pi\pi_t$. The case when the environmental protection function is given by $\Pi\pi_{1t}^{\beta}\pi_{2t}^{1-\beta}$ can be analyzed as before and will not be pursued here. The interior solution to the social planner's problem gives:

$$E_t = E_0 e^{\frac{\varphi}{\eta}t},$$

$$c_t = \left(\frac{E_0^{\eta}}{\tilde{\phi}}\right)^{1/\sigma} e^{\frac{\varphi}{\sigma}t},$$

$$K_t = \frac{N\sigma}{\sigma(\varphi+\rho)-\varphi}\left(\frac{E_0^{\eta}}{\tilde{\phi}}\right)^{1/\sigma} e^{\frac{\varphi}{\sigma}t} + \left(\frac{\varphi}{\eta} - \xi\right)\frac{\eta}{\eta(\varphi+\rho)-\varphi}\frac{E_0}{\Pi}e^{\frac{\varphi}{\eta}t},$$

$$\pi_t = \left[1 + \frac{P}{\Pi}\frac{\eta}{[\eta(\varphi+\rho)-\varphi]}\right]\left(\frac{\varphi}{\eta} - \xi\right)\frac{E_0}{\Pi}e^{\frac{\varphi}{\eta}t} + \frac{P}{\Pi}\frac{N\sigma}{[\sigma(\varphi+\rho)-\varphi]}\left(\frac{E_0^{\eta}}{\tilde{\phi}}\right)e^{\frac{\varphi}{\sigma}t},$$

where

$$\frac{1}{\tilde{\phi}} \equiv \frac{c_t^{\sigma}}{E_t^{\eta}} = \left(A - \frac{P}{\Pi} - \delta - \xi\right)\frac{\Psi}{N\Lambda\Pi},$$

and we assume that σ, η, φ, ρ and ξ are such that all the variables of the model are positive and increase along the optimal path.

Furthermore, the ratio of capital to environmental quality is given by:

$$\frac{K_t}{E_t} = \frac{N\sigma}{\sigma(\varphi+\rho)-\varphi}\left(\frac{E_0^{\eta}}{\tilde{\phi}}\right)^{1/\sigma}\frac{1}{E_0}e^{\left(\frac{1}{\sigma}-\frac{1}{\eta}\right)\varphi t} + \frac{1}{\Pi}\left(\frac{\varphi}{\eta} - \xi\right)\frac{\eta}{\eta(\varphi+\rho)-\varphi}. \tag{21}$$

Figure 3: [Approximately Here.]

Therefore, the optimal path for the economy results only if at time zero the ratio of capital to environmental quality is constant and equal to:

$$\frac{K_0}{E_0} = \frac{N\sigma}{\sigma(\varphi+\rho)-\varphi}\left(\frac{E_0^{\eta}}{\tilde{\phi}}\right)^{1/\sigma}\frac{1}{E_0} + \frac{1}{\Pi}\left(\frac{\varphi}{\eta}-\xi\right)\frac{\eta}{\eta(\varphi+\rho)-\varphi}.$$

As in the previous case, this is not true in general, and the U-shaped (or more closely $\mathcal{V}$-shaped) path for environmental quality results from the social planner's problem if environmental quality is abundant at time zero. Expenditures on environmental protection are equal to zero from time zero to t_1 and positive and increasing afterwards.

It is clear from the above equations that balanced growth will result if and only if the elasticity of marginal utility of consumption is equal to the elasticity of marginal utility of environmental quality, i.e., $\sigma = \eta$. If this is the case, constant taxes on pollution and consumption implement the social optimum in the decentralized economy after the unregulated era (time zero to t_1), implying a constant rate of return to capital:

$$\tau_p = \frac{1}{\Pi},$$

$$\tau_c = \frac{\tilde{\phi}^{1/\sigma}}{N\Pi}\left(\frac{\varphi}{\eta}-\xi\right),$$

and

$$r = A - \frac{P}{\Pi} - \delta.$$

If $\sigma \neq \eta$, then the ratio of capital to environmental quality either increases or decreases in the regulated decentralized economy depending on whether $\sigma \lesseqgtr \eta$. Furthermore, we cannot explicitly solve for the time variant tax rate on consumption τ_c, and the tax rate on pollution and the rental rate of capital change with time as follows:

$$\tau_p = \frac{1}{\Pi} - \frac{1}{P}\frac{\dot{\tau}_c}{(1+\tau_c)},$$

and

$$r = A - \frac{P}{\Pi} - \delta + \frac{\dot{\tau}_c}{(1+\tau_c)}.$$

Figure 3 shows the optimal paths for the ratio of capital to environmental quality depending on the elasticity of marginal utility of consumption and environmental quality. Either case produces decreasing environmental quality from time zero to t_1, and increasing afterwards. After time t_1, when the interior solution applies, the ratio of capital to environmental quality asymptotically approaches a constant (second term in equation (21)) if $\sigma > \eta$, increases if $\sigma < \eta$, or remains constant if $\sigma = \eta$.

IX CONCLUSION

This paper explores the underlying causes of the relationship between economic growth and environmental quality. The model is distinctive in its simplicity, explicit recognition of environmental quality as a public good whose optimal provision goes beyond pollution control, and freedom from institutional details, intergenerational conflicts, and assumptions about technology and the pollution intensity of productive capital.

Pareto optimality in the model drives three important results: (i) environmental quality decreases at early stages of development at increasing rates but eventually starts to increase; (ii) environmental protection expenditures are negligible or absent at early stages of development, when capital accumulation is more crucial to economic growth; and (iii) environmental protection is pursued at later stages of development and causes the rate of economic growth to fall. In an economy with an initially abundant stock of environmental quality, capital initially accumulates relatively fast and environmental quality declines at an increasing rate. However, once the optimal steady state ratio of capital to environmental quality is realized, more resources are devoted to environmental protection and environmental quality starts to improve. The eventual shift toward environmental protection reduces the rate of economic growth and is reflected by a smaller rate of growth of consumption and capital, and a smaller rate of decay in the shadow value of capital.

Moreover, the analysis of the decentralized economy sheds light into the design of policies to implement the social optimum. We show how traditional instruments such as a pollution tax or tradable pollution permits when implemented without other instruments can induce the optimal level of pollution, but fail to endorse the optimal provision of environmental quality. The intuition is that the government can further engage pollution abatement, recovery of the degraded environment and promotion of other improvements, thus optimally providing the public good environmental quality. A pollution tax implemented without a simultaneous consumption tax will not generate enough revenues for the government to optimally provide environmental quality. Furthermore, tradable pollution permits will only induce the social optimum if they are sold for the same price as the pollution tax, and consumption is simultaneously taxed. Freely distributed tradable pollution permits need to be coupled with taxes on consumption and profits.

This paper also solves for the time when environmental quality starts to improve in a decentralized economy and analyzes its determinants. In particular, we show how the efficiency of expenditures on environmental protection can explain why different environmental problems exhibit different turning points for environmental quality. Furthermore, we show that optimality of the EKC is robust to a more general class of utility functions with constant elasticity of marginal utility.

A Appendix A

The Lagrangian for the maximization problem is: $\mathcal{L}_t = N[\alpha \ln(c_t) + (1-\alpha)\ln(E_t)] + \lambda_t[-PK_t + \Pi\pi_t + \xi E_t] + \mu_t[AK_t - Nc_t - \pi_t - \delta K_t] + \theta_t \pi_t$ Setting $\frac{\partial \mathcal{L}_t}{\partial c_t} = 0$ and $\frac{\partial \mathcal{L}_t}{\partial \pi_t} = 0$, yields equations (1) and (2). Equations (3) and (4) are obtained by setting $\dot{\lambda}_t = \rho\lambda_t - \frac{\partial \mathcal{L}_t}{\partial E_t}$ and $\dot{\mu}_t = \rho\mu_t - \frac{\partial \mathcal{L}_t}{\partial K_t}$.

Equation (4) is a differential equation for μ_t with solution:

$$\mu_t = e^{\left(\frac{P}{\Pi}+\rho+\delta-A\right)t}\left(\tilde{\mu} - \frac{P}{\Pi}\int_0^t e^{-\left(\frac{P}{\Pi}+\rho+\delta-A\right)s}\theta_s ds\right)$$

where $\tilde{\mu}$ is an arbitrary constant.

We can rewrite the transversality condition as in (6):

$$\lim_{t\to\infty} e^{-\rho t} e^{\left(\frac{P}{\Pi}+\rho+\delta-A\right)t}\left(\tilde{\mu} - \frac{P}{\Pi}\int_0^t e^{-\left(\frac{P}{\Pi}+\rho+\delta-A\right)s}\theta_s ds\right) K_t = 0$$

To obtain the differential equation (7) for consumption, differentiate equation (1) with respect to time and set it equal to equation (4), making use of equation (1) to substitute for μ_t and the fact that in the steady state an interior solution implies that $\theta_t = 0$:

$$-\frac{\alpha}{c_t}\frac{\dot{c}_t}{c_t} = \frac{\alpha}{c_t}\left(\frac{P}{\Pi} + \rho + \delta - A\right)$$

$$\frac{\dot{c}_t}{c_t} = \left(A - \frac{P}{\Pi} - \rho - \delta\right) \tag{I}$$

Equation (I) is an autonomous ODE with solution:

$$c_t = \tilde{c}e^{\varphi t} \tag{II}$$

where $\tilde{c}$ is a constant to be determined and $\varphi \equiv \left(A - \frac{P}{\Pi} - \rho - \delta\right)$.

Next, differentiate (2) with respect to time and set it equal to (3), making use of equation (1) to substitute for μ_t:

$$-\frac{1}{\Pi}\frac{\alpha}{c_t}\frac{\dot{c}_t}{c_t} = (\rho - \xi)\frac{1}{\Pi}\frac{\alpha}{c_t} - N\frac{(1-\alpha)}{E_t} \therefore$$

$$\frac{\dot{c}_t}{c_t} = N\Pi\frac{(1-\alpha)}{\alpha}\frac{c_t}{E_t} - \rho + \xi \tag{III}$$

Setting (I) equal to (III) yields:

$$\frac{c_t}{E_t} = \left(A - \frac{P}{\Pi} - \delta - \xi\right)\frac{1}{\Pi}\frac{\alpha}{(1-\alpha)}\frac{1}{N}$$

Or in a more simplified notation:

$$E_t = \phi c_t \tag{IV}$$

Equation (IV) must be satisfied for an optimal solution for the social planner's problem. Thus, from (II) and (IV), we can derive the time path for environmental quality:

$$E_t = \phi c_t = \phi\tilde{c}e^{\varphi t} \tag{V}$$

But, since E_0 is given, equations (II) and (V) become:

$$E_t = E_0 e^{\varphi t} \tag{VI}$$

$$c_t = \frac{E_0}{\phi}e^{\varphi t} \tag{VII}$$

To derive the equations for K_t and π_t, first differentiate (VI) with respect to time, set it equal to the equation of motion for environmental quality and solve for π_t:

$$E_0\varphi e^{\varphi t} = -PK_t + \Pi\pi_t + \xi E_0 e^{\varphi t} \therefore$$

$$\pi_t = \frac{E_0 e^{\varphi t}(\varphi - \xi) + PK_t}{\Pi} \tag{VIII}$$

Next, substitute (VII) and (VIII) into the equation of motion for the capital stock to obtain a differential equation for K_t and its respective solution:

$$\dot{K}_t = AK_t - N\frac{E_0}{\phi}e^{\varphi t} - \frac{E_0 e^{\varphi t}(\varphi - \xi)}{\Pi} - \frac{P}{\Pi}K_t - \delta K_t \therefore$$

$$\dot{K}_t - \left(A - \frac{P}{\Pi} - \delta\right) K_t = -\left(\frac{N}{\phi} + \frac{(\varphi - \xi)}{\Pi}\right) E_0 e^{\varphi t} \therefore$$

$$K_t = \left(\frac{N}{\phi} + \frac{(\varphi - \xi)}{\Pi}\right) \frac{E_0}{\rho} e^{\varphi t} + \tilde{K} e^{(\varphi + \rho)t} \quad \text{(IX)}$$

To determine the constant $\tilde{K}$, plug equation (IX) into the transversality condition (6), recalling that for an interior solution $\theta_t = 0$:

$$\lim_{t \to \infty} e^{-\rho t} \, \tilde{\mu} \, e^{-\varphi t} \left[\left(\frac{N}{\phi} + \frac{(\varphi - \xi)}{\Pi}\right) \frac{E_0}{\rho} e^{\varphi t} + \tilde{K} e^{(\varphi + \rho)t}\right] = 0 \therefore$$

$$\lim_{t \to \infty} \tilde{\mu} \left[\left(\frac{N}{\phi} + \frac{(\varphi - \xi)}{\Pi}\right) \frac{E_0}{\rho} e^{-\rho t} + \tilde{K}\right] = 0 \quad \text{(X)}$$

It follows from (X) that the transversality condition will hold if and only if $\tilde{K} = 0$. Therefore, as in (10), the equation for K_t becomes

$$K_t = \left(\frac{N}{\phi} + \frac{(\varphi - \xi)}{\Pi}\right) \frac{E_0}{\rho} e^{\varphi t},$$

which reduces to:

$$K_t = \frac{(\varphi - \xi + \alpha\rho)}{(1 - \alpha)\rho\Pi} E_0 e^{\varphi t}, \quad \text{(XI)}$$

Plugging (XI) into (VIII), we obtain the equation (11) for π_t:

$$\pi_t = \left[\frac{(\varphi - \xi)}{\Pi} + \frac{P}{\Pi} \frac{(\varphi - \xi + \alpha\rho)}{(1 - \alpha)\rho\Pi}\right] E_0 e^{\varphi t}. \quad \text{(XII)}$$

To derive equations (14)–(16) for c_t, K_t and E_t during the transition to the steady state, first differentiate equation (1) with respect to time and substitute the result into equation (4):

$$\dot{\mu}_t = -\frac{\alpha \dot{c}_t}{c_t^2},$$

$$-\frac{\alpha \dot{c}_t}{c_t^2} = \frac{\alpha}{c_t}\left(\frac{P}{\Pi} + \rho + \delta - A\right) - \theta_t \frac{P}{\Pi}.$$

Rearranging, we obtain:

$$\dot{c}_t = \left(A - \frac{P}{\Pi} - \rho - \delta\right) c_t + \theta_t \frac{P}{\Pi\alpha} c_t^2. \quad \text{(XIII)}$$

Equation (XIII) is a Bernoulli differential equation with $n = 2$. To solve that equation, rewrite it as:

$$\frac{\dot{c}_t}{c_t} = \varphi + \theta_t \frac{P}{\Pi\alpha} c_t \quad \text{(XIV)}$$

Next, define $v_t = \frac{1}{c_t}$, so that $\dot{v}_t = -\frac{\dot{c}_t}{c_t^2}$, and divide both sides of (XIV) by c_t:

$$\frac{\dot{c}_t}{c_t^2} = \varphi \frac{1}{c_t} + \theta_t \frac{P}{\Pi\alpha} \quad \text{(XV)}$$

Substitute v_t for $\frac{1}{c_t}$ in equation (XV) and rearrange to obtain:

$$\dot{v}_t + \varphi v_t = -\theta_t \frac{P}{\Pi\alpha} \tag{XVI}$$

Equation (XVI) is an ODE with solution:

$$v_t = e^{-\varphi t}\left(\tilde{v} - \frac{P}{\Pi\alpha}\int_0^t \theta_s e^{\varphi s} ds\right) \tag{XVII}$$

Where $\tilde{v}$ is a constant. To obtain equation (14), substitute c_t for $\frac{1}{v_t}$ in equation (XVII), use the initial condition c_0, and rearrange. Finally, equations (15) and (16) are obtained by solving the differential equations $\dot{K}_t = AK_t - Nc_t - \delta K_t$ and $\dot{E}_t = -PK_t + \xi E_t$, and using the initial conditions K_0 and E_0.

If the environmental protection function is given by $\Pi\pi_{1t}^{\beta}\pi_{2t}^{1-\beta}$, with $0 < \beta < 1$, the current value dynamic Lagrangian and the first order conditions become

$$\mathcal{L} = N[\alpha\ln(c_t) + (1-\alpha)\ln(E_t)] + \lambda_t[-PK_t + \Pi\pi_{1t}^{\beta}\pi_{2t}^{1-\beta} + \xi E_t] + \mu_t[AK_t - Nc_t - \pi_{1t} - \pi_{2t} - \delta K_t],$$

$$\mu_t = \frac{\alpha}{c_t},$$

$$\lambda_t = \frac{(\mu_t - \theta_{1t})}{\beta\Pi}\left(\frac{\pi_{2t}}{\pi_{1t}}\right)^{\beta-1},$$

$$\lambda_t = \frac{(\mu_t - \theta_{2t})}{(1-\beta)\Pi}\left(\frac{\pi_{2t}}{\pi_{1t}}\right)^{\beta},$$

$$\dot{\lambda}_t = \lambda_t(\rho - \xi) - N\frac{(1-\alpha)}{E_t},$$

$$\dot{\mu}_t = \mu_t\left[\frac{P}{(1-\beta)\Pi}\left(\frac{\pi_{2t}}{\pi_{1t}}\right)^{\beta} + \rho + \delta - A\right] - \frac{P\theta_{1t}}{(1-\beta)\Pi}\left(\frac{\pi_{2t}}{\pi_{1t}}\right)^{\beta}.$$

The first order conditions define the optimal ratio of the two types of expenditures in an interior solution:

$$\frac{\pi_{1t}}{\pi_{2t}} = \frac{\beta}{1-\beta}.$$

Usual manipulations of the first order conditions give the remaining results for the balanced growth path:

$$\frac{1}{\hat{\phi}} \equiv \frac{c_t}{E_t} = \left[A - \frac{P}{(1-\beta)\Pi}\left(\frac{1-\beta}{\beta}\right)^{\beta} - \delta - \xi\right]\frac{\alpha}{N\Pi(1-\alpha)\beta}\left(\frac{1-\beta}{\beta}\right)^{\beta-1},$$

$$\hat{\varphi} = \left[A - \frac{P}{(1-\beta)\Pi}\left(\frac{1-\beta}{\beta}\right)^{\beta} - \rho - \delta\right],$$

$$E_t = E_0 e^{\hat{\varphi}t},$$

$$c_t = \frac{E_0}{\hat{\phi}} e^{\hat{\varphi}t},$$

$$K_t = \frac{(\hat{\varphi} - \xi + \alpha\rho)}{(1-\alpha)\rho\Pi} \left(\frac{1-\beta}{\beta}\right)^{\beta} \frac{1}{1-\beta} E_0 e^{\hat{\varphi}t},$$

$$\pi_{1t} = \left[\frac{(\hat{\varphi} - \xi)}{\Pi} + \frac{P}{\Pi} \frac{(\hat{\varphi} - \xi + \alpha\rho)}{(1-\alpha)\rho\Pi} \left(\frac{1-\beta}{\beta}\right)^{\beta} \frac{1}{1-\beta}\right] \left(\frac{\beta}{1-\beta}\right)^{1-\beta} E_0 e^{\hat{\varphi}t},$$

$$\pi_{2t} = \left(\frac{1-\beta}{\beta}\right) \pi_{1t}.$$

B Appendix B

First consider a static version of the problem in section I. The social planner maximizes the utility of the N identical individuals subject to the equation for environmental quality and the social feasibility constraint:

$$\max_{c,\pi} N[\alpha \ln(c) + (1-\alpha)\ln(E)],$$

subject to

$$E = \bar{E} - PK + \Pi\pi - \xi E,$$

and

$$AK \geq Nc + \pi + \delta K,$$

where the new term $\bar{E}$ indicates the endowment of environmental quality, and the stock of capital K is given. The first order conditions with respect to c and π, and equality in the feasibility constraint give the optimal consumption and environmental expenditures:

$$c = \frac{\alpha}{N}\left[(A-\delta)K + \frac{(\bar{E} - PK)}{\Pi}\right],$$

$$\pi = (1-\alpha)(A-\delta)K - \alpha\frac{(\bar{E} - PK)}{\Pi}.$$

It is easy to verify that the optimum is implemented with a pollution tax in a decentralized economy where the households' problem is

$$\max_{c} N[\alpha \ln(c) + (1-\alpha)\ln(E),]$$

subject to

$$rK \geq Nc,$$

where r is the rental price of capital; and the firm's problem is

$$\max_K AK - rK - \delta K - \tau_p PK,$$

where τ_p is the pollution tax. The optimal pollution tax and rental price of capital are:

$$\tau_p = (1-\alpha)\frac{(A-\delta)}{P} - \alpha\frac{(\bar{E} - PK)}{\Pi PK},$$

$$r = \alpha\left[(A-\delta) + \frac{(\bar{E} - PK)}{\Pi K}\right].$$

On the other hand, if the only instrument the government uses in a dynamic economy is a tax on pollution, the households' problem becomes:

$$\max_{c_t} \int_0^\infty e^{-\rho t} N[\alpha \ln(c_t) + (1-\alpha)\ln(E_t)]\, dt$$

subject to

$$\dot{K}_t = rK_t - Nc_t,$$

and the transversality condition

$$\lim_{t\to\infty} e^{-\rho t}\nu_t K_t = 0.$$

The first order conditions imply:

$$\frac{\dot{\nu}_t}{\nu_t} = \rho - r,$$

and

$$\frac{\dot{c}_t}{c_t} = r - \rho.$$

The firms' problem is

$$\max_{K_t} AK_t - rK_t - \delta K_t - \tau_p PK_t,$$

and the first order condition gives the competitive price of capital:

$$r = A - \delta - \tau_p P.$$

Consumption will grow at the optimal rate if and only if

$$\tau_p = \frac{1}{\Pi},$$

implying that the rental price of capital is given by

$$r = A - \delta - \frac{P}{\Pi} = \varphi + \rho,$$

and the consumption path, after choosing the initial consumption level according to the social planner's solution, is

$$c_t = \frac{E_0}{\phi} e^{\varphi t}.$$

To solve for K_t, use the equation of motion for capital, the transversality condition and the equation for c_t to obtain

$$K_t = \frac{N}{\rho}\frac{E_0}{\phi} e^{\varphi t} = \frac{(\varphi - \xi + \rho)\alpha}{(1-\alpha)\rho\Pi} E_0 e^{\varphi t},$$

which is smaller than K_t in equation (10)[7]. Therefore, less will be produced at each time t than socially desirable, and since consumption is the same as in the social planner's solution, less will be left for environmental expenditures. To see that, compute the resulting tax revenues:

$$TR_t = \tau_p P K_t = \frac{P}{\Pi}\frac{(\varphi - \xi - \rho)\alpha}{(1-\alpha)\rho\Pi} E_0 e^{\varphi t} < \pi_t.$$

By solving the differential equation for environmental quality using tax revenues to provide environmental quality

$$\dot{E}_t = -PK_t + \Pi TR_t + \xi E_t,$$

we obtain the equation for E_t, which grows at a slower rate than in the social planner's solution:

$$E_t = E_0 e^{\xi t}.$$

Instead, we can choose the constant of integration for consumption to yield the optimal level of the stock of capital at each time t, but it is easy to show that the resulting consumption path will be above the social planner's optimum and less resources than desirable will be left for environmental protection. As before, environmental quality will grow at its natural rate of recovery.

Next, consider a tax on cumulative pollution at time t. The firm's problem becomes

$$\max_{K_t} AK_t - rK_t - \delta K_t - \tau_p \int_0^t PK_s ds,$$

and the zero-profit condition implies that the rental price of capital is

$$r = A - \delta - \tau_p P \frac{\int_0^t K_s ds}{K_t}.$$

[7]To see that, subtract the previous expression from the right hand side in equation (10), making use of the assumption that $\varphi - \xi > 0$.

Consumption will grow at the optimal rate if and only if

$$\tau_p = \frac{1}{\Pi} \frac{K_t}{\int\limits_0^t K_s ds},$$

implying that the rental price of capital is

$$r = A - \frac{P}{\Pi} - \delta = \varphi + \rho,$$

and consumption is given by

$$c_t = \frac{E_0}{\phi} e^{\varphi t}.$$

As before, capital at time t differs from the socially optimal capital stock and tax revenues are insufficient to cover the optimal expenditures on environmental protection:

$$K_t = \frac{N}{\rho} \frac{E_0}{\phi} e^{\varphi t},$$

$$TR_r = \tau_p P K_t < \pi_t.$$

Lastly, if the optimal taxes on pollution and consumption are implemented with an environmental protection function given by $\Pi \pi_{1t}^{\beta} \pi_{2t}^{1-\beta}$, the pollution tax is

$$\hat{\tau_p} = \frac{1}{\Pi(1-\beta)} \left(\frac{1-\beta}{\beta} \right)^{\beta},$$

the rental price of capital is

$$\hat{r} = A - \delta - \frac{P}{\Pi(1-\beta)} \left(\frac{1-\beta}{\beta} \right)^{\beta},$$

and the tax on consumption is

$$\hat{\tau_c} = \frac{\hat{\phi}(\hat{\varphi} - \xi)}{N\Pi(1-\beta)} \left(\frac{1-\beta}{\beta} \right)^{\beta}.$$

The results for the freely distributed pollution permits are the same with $\hat{\tau_\eta} = 1$.

C Appendix C

Proposition 2 *Starting with abundant environmental quality, the approach to the optimal ratio of capital to environmental quality without government intervention (no taxes) is Pareto optimal.*

Proof:

Without government intervention, no taxes are present in the households' or firm's problems. During the transition, the households' problem is

$$\max_{c_t} \int_0^\infty e^{-\rho t} N[\alpha \ln(c_t) + (1-\alpha)\ln(E_t)]\, dt$$

subject to

$$\dot{K}_t = rK_t - Nc_t,$$

and the transversality condition

$$\lim_{t\to\infty} e^{-\rho t}\nu_t K_t = 0.$$

The first order conditions give the rate of growth of consumption:

$$\frac{\dot{c}_t}{c_t} = r - \rho.$$

Firms maximize profits and the resulting rental price of capital is $r = A - \delta$. In equilibrium, the transversality condition and the initial stock of capital yield the equations for consumption per capita

$$c_t = \frac{\rho K_0}{N} e^{(A-\delta-\rho)t}, \tag{XVIII}$$

and capital

$$K_t = K_0 e^{(A-\delta-\rho)t}. \tag{XIX}$$

Environmental quality decreases with the stock of capital and increases at its own natural rate of recovery:

$$\dot{E}_t = -PK_t + \xi E_t.$$

Solving for E_t with E_0 given, we have

$$E_t = -\frac{P}{(A-\delta-\rho-\xi)} K_0 e^{(A-\delta-\rho)t} + \left[E_0 + \frac{PK_0}{(A-\delta-\rho-\xi)}\right] e^{\xi t}. \tag{XX}$$

We need to show that equations (XVIII), (XIX) and (XX) correspond to equations (14), (15) and (16). To see that this is the case, choose $\theta_t = \mu_t$, which from the first order condition (2) from section I implies that the shadow value of the environment during the transition is zero. That is, the amount of resources society is willing to sacrifice for an additional unit of environmental quality when it is abundant is zero. Furthermore, from equation (4), the shadow value of capital becomes:

$$\mu_t = \theta_t = \tilde{\mu} e^{-(A-\delta-\rho)t}.$$

Plugging this result into equation (14) and solving for the integral in the denominator, we obtain:

$$c_t = \frac{c_0 e^{\varphi t}}{1 + \frac{c_0}{\alpha}\left(\tilde{\mu} e^{-\frac{P}{\Pi}t} - \frac{P}{\Pi}\tilde{c}\right)},$$

where $\tilde{c}$ is a constant of integration. Next, let $c_0 = \frac{\rho K_0}{N}$ and choose $\tilde{c}$ such that $c_t = c_0 e^{(A-\delta-\rho)t}$, so that the Pareto optimal equation for c_t coincides with the decentralized solution without government intervention:

$$\frac{\rho K_0}{N} e^{(A-\delta-\rho)t} = \frac{\frac{\rho K_0}{N} e^{\varphi t}}{1 + \frac{\rho K_0}{N\alpha}\left(\tilde{\mu} e^{-\frac{P}{\Pi}t} - \frac{P}{\Pi}\tilde{c}\right)}.$$

Solving for $\tilde{c}$, we can see that $\tilde{c}$ is a constant if and only if $\tilde{\mu} = \frac{N\alpha}{\rho K_0}$. This implies that c_t is given by equation (XVIII).

Next, we plug the value for c_t derived above into equation (15), describing the Pareto optimal path for capital during the transition. Solving for K_t, we obtain:

$$K_t = K_0 e^{(A-\delta-\rho)t} + [-N\tilde{K} + K_0] e^{(A-\delta)t},$$

where $\tilde{K}$ is a constant of integration. Choose $\tilde{K} = \frac{K_0}{N}$ and we obtain equation (XIX).

Finally, plug the result for K_t into equation (16) and solve for E_t:

$$E_t = -\frac{PK_0}{(A-\delta-\rho-\xi)} e^{(A-\delta-\rho)t} + [E_0 - P\tilde{E}] e^{\xi t},$$

where $\tilde{E}$ is a constant of integration. Since E_0 is given, $\tilde{E} = -\frac{K_0}{(A-\delta-\rho-\xi)}$, which implies that E_t is given by equation (XX). ■

During the transition from abundant environmental quality to balanced growth in the decentralized economy, the stock of capital, environmental quality and the ratio of capital to the environment are:

$$K_t = K_0 e^{(A-\delta-\rho)t},$$

$$E_t = -\frac{PK_0}{(A-\delta-\rho-\xi)} e^{(A-\delta-\rho)t} + \left[E_0 + \frac{PK_0}{(A-\delta-\rho-\xi)}\right] e^{\xi t},$$

and

$$\frac{K_t}{E_t} = \frac{K_0 e^{(A-\delta-\rho)t}}{-\frac{PK_0}{(A-\delta-\rho-\xi)} e^{(A-\delta-\rho)t} + \left[E_0 + \frac{PK_0}{(A-\delta-\rho-\xi)}\right] e^{\xi t}}. \tag{XXI}$$

From proposition 1 at time t_1, the ratio of capital to environmental quality is equal to the optimal ratio of capital to environmental quality R given by equation (12). Setting (XXI) equal to (12) at time t_1 we can calculate the transition time $t1$:

$$\frac{K_{t_1}}{E_{t_1}} = \frac{K_0 e^{(A-\delta-\rho)t_1}}{-\frac{PK_0}{(A-\delta-\rho-\xi)} e^{(A-\delta-\rho)t_1} + \left[E_0 + \frac{PK_0}{(A-\delta-\rho-\xi)}\right] e^{\xi t_1}} = R \therefore$$

$$-\frac{P}{(A-\delta-\rho-\xi)} + \frac{E_0(A-\delta-\rho-\xi) + PK_0}{K_0(A-\delta-\rho-\xi)} e^{-(A-\delta-\rho-\xi)t_1} = \frac{1}{R} \therefore$$

$$-(A-\delta-\rho-\xi)t_1 = \ln\left[\left[\frac{1}{R} + \frac{P}{(A-\delta-\rho-\xi)}\right] \frac{K_0(A-\delta-\rho-\xi)}{E_0(A-\delta-\rho-\xi) + PK_0}\right] \therefore$$

$$t_1 = -\frac{1}{(A-\delta-\rho-\xi)} \ln\left[\frac{K_0(A-\delta-\rho-\xi) + K_0PR}{E_0(A-\delta-\rho-\xi)R + K_0PR}\right].$$

To have $t_1 > 0$ we need the term in brackets to be less than 1, so that its natural logarithm is negative. Since the second term in the numerator of the expression in brackets is identical to the second term in the denominator, it suffices to show that the first term in the numerator is smaller than the first term in the denominator. This follows from abundant environmental quality at time zero:

$$\frac{K_0}{E_0} < R \quad \Rightarrow \quad K_0 < E_0 R \quad \Rightarrow \quad K_0(A-\delta-\rho-\xi) < E_0(A-\delta-\rho-\xi)R.$$

The effect of α, E_0, K_0 and P on t_1 are:

$$\frac{\partial t_1}{\partial \alpha} = \frac{\frac{\partial R}{\partial \alpha}}{(A-\delta-\rho-\xi+PR)R} = \frac{\varphi+\rho-\xi}{(A-\delta-\rho-\xi+PR)R(1-\alpha)^2\rho\Pi} > 0,$$

$$\frac{\partial t_1}{\partial E_0} = \frac{1}{E_0(A-\delta-\rho-\xi) + K_0P} > 0,$$

$$\frac{\partial t_1}{\partial K_0} = -\frac{E_0}{K_0[E_0(A-\delta-\rho-\xi) + K_0P]} < 0,$$

$$\frac{\partial t_1}{\partial P} = -\frac{R(E_0R - K_0) - [E_0(A-\delta-\rho-\xi) + K_0P]\frac{\partial R}{\partial P}}{R(A-\delta-\rho-\xi+PR)[E_0(A-\delta-\rho-\xi) + K_0P]} =$$

$$-\frac{R(E_0R - K_0) + \frac{[E_0(A-\delta-\rho-\xi)+K_0P]}{(1-\alpha)\rho\Pi^2}}{R(A-\delta-\rho-\xi+PR)[E_0(A-\delta-\rho-\xi) + K_0P]} < 0,$$

where abundant environmental quality at time zero is defined as $E_0R > K_0$, so that the first term in the numerator of the last two expressions is positive.

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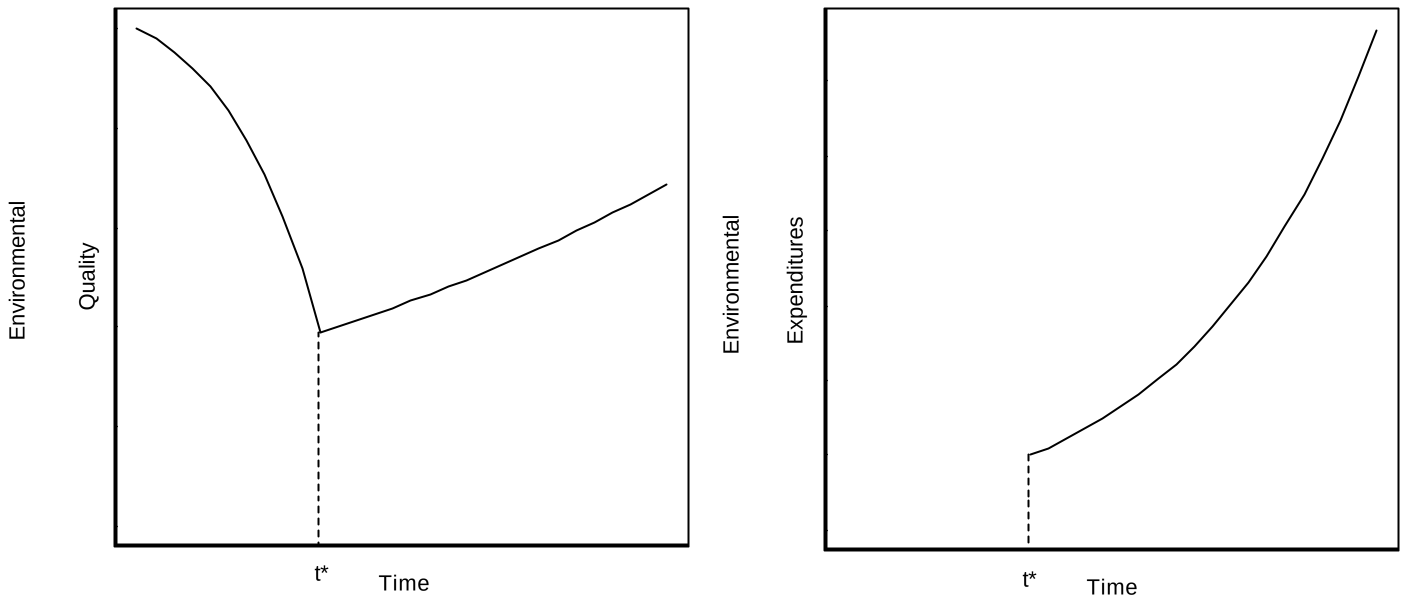

Figure 1: Environmental Quality and Environmental Expenditures

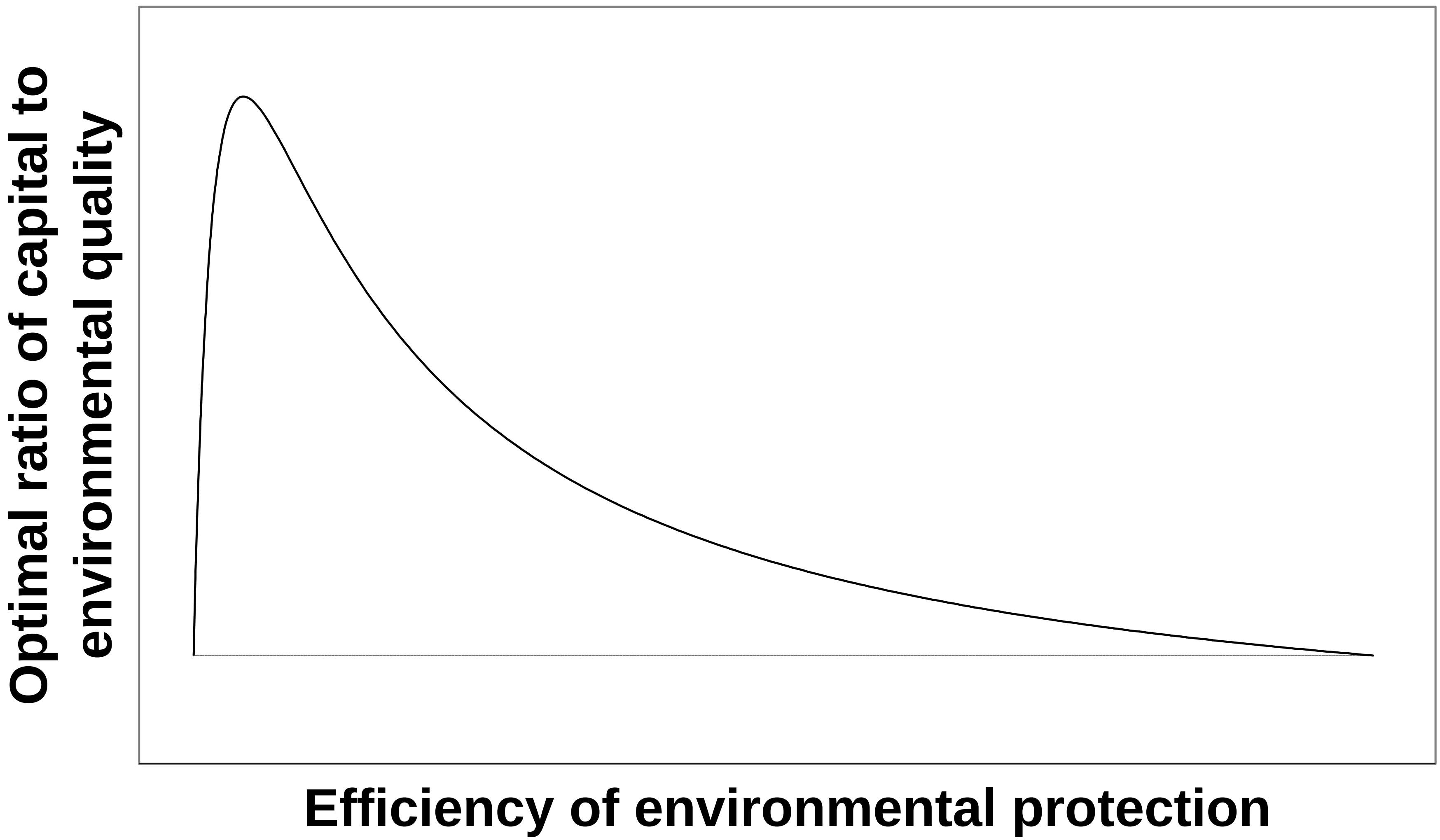

Figure 2(a): Efficiency of Environmental Protection and (K/E)

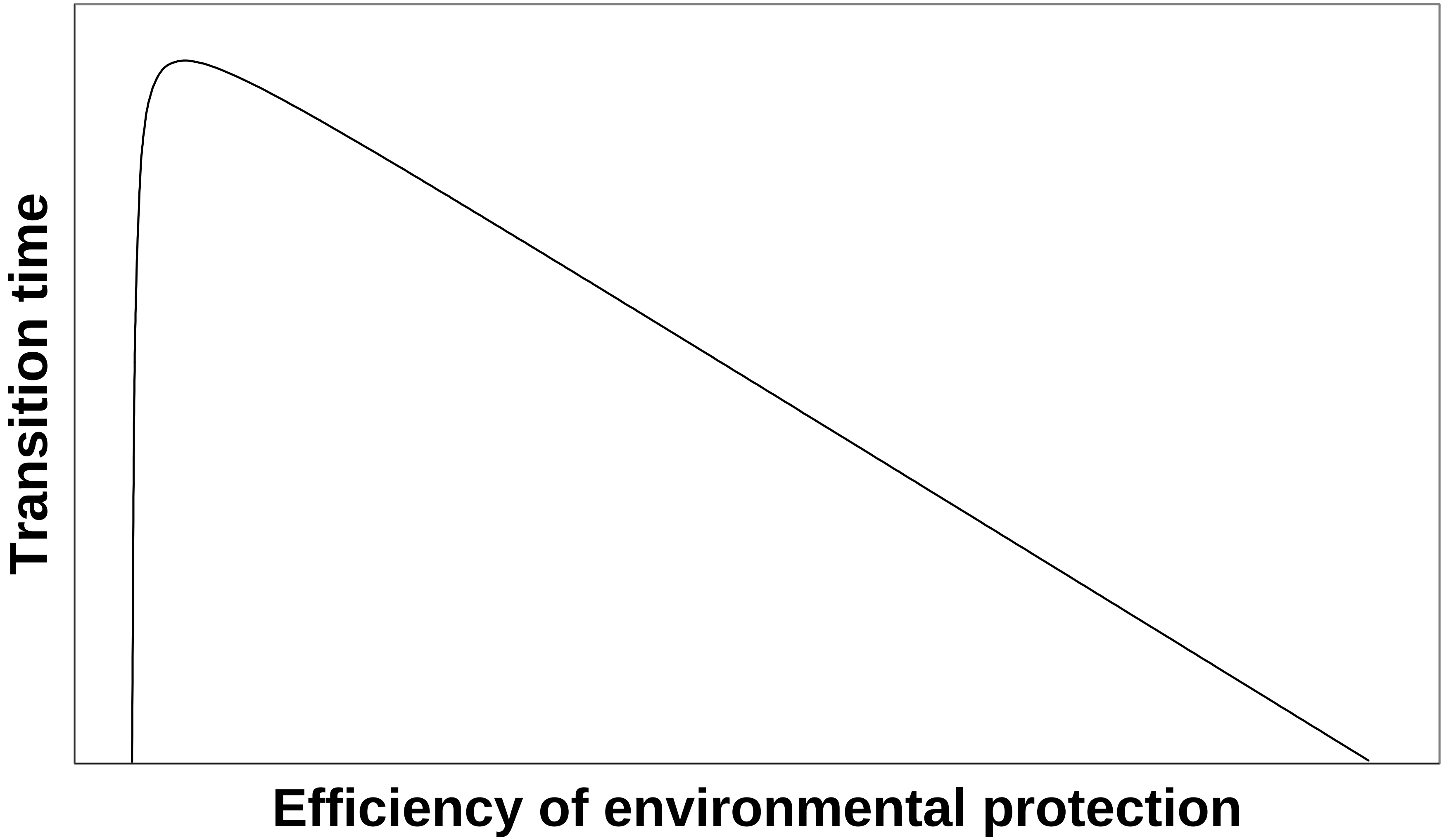

Figure 2(b): Efficiency of Environmental Protection and t1

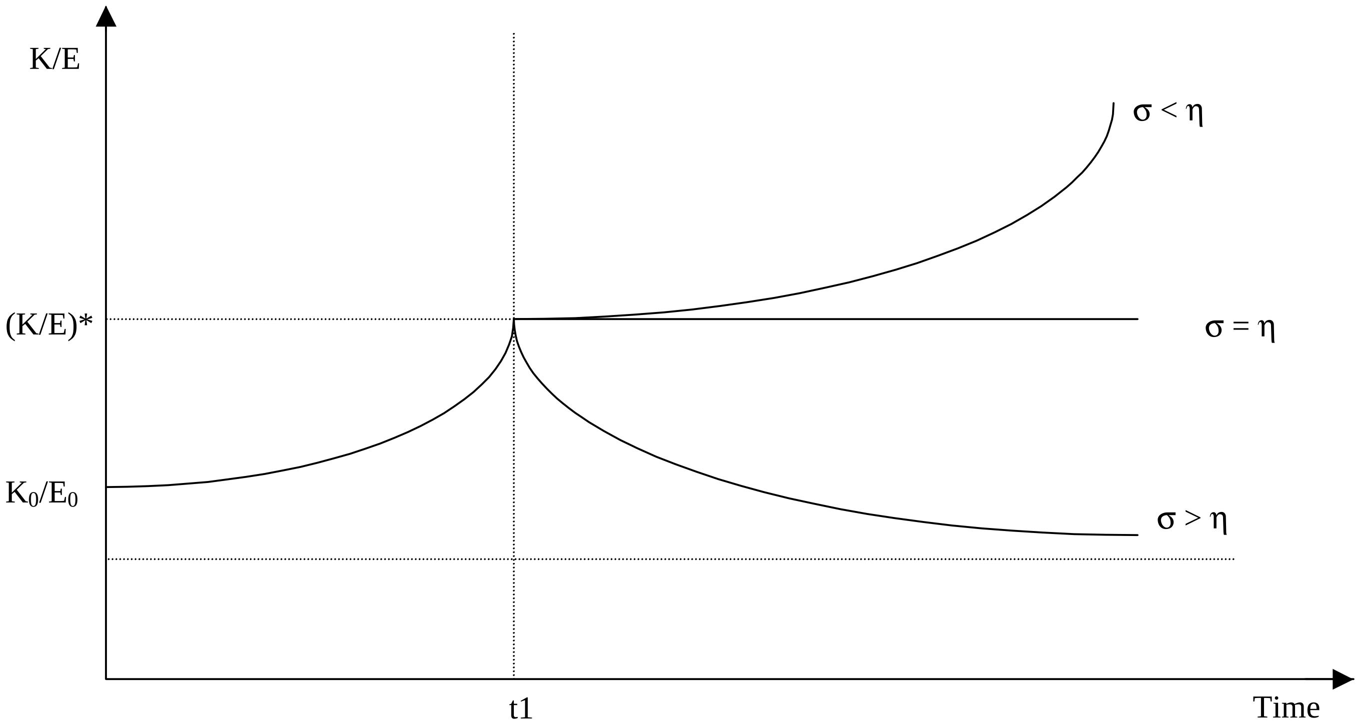

Figure 3: Elasticity of Marginal Utility and Optimal Ratio of Capital to Environmental Quality.